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Student Teachers' Interactive Decisions with Respect to Student Mathematics Thinking

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ABSTRACT

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Teaching mathematics is a difficult and complicated task. For student teachers, who are extremely new to the mathematics classroom, this difficulty is magnified. One of the biggest challenges for student teachers is learning how to effectively use the student thinking that emerges during mathematics lessons. I report the results of a case study of two mathematics education student teachers. I focus on how they make decisions while teaching in order to use their students' mathematical thinking. I also present analysis of the student teachers' discourse patterns, the reasons they gave to justify these patterns, and how their reasons affected how they used their students' thinking. I found that generally the student teachers used student thinking in ineffective ways. However, the reasons the student teachers gave for using student thinking always showed the best of intentions. Though given with the best of intentions, most of the reasons for using student thinking given by the student teachers were correlated with the student teachers ineffectively using their student's thinking. However, some of the reasons given by the STs for using student thinking seemed to help the student teachers more effectively use their students' thinking. I conclude with implications for preparing future student teachers to better use student thinking.

Keywords: discourse analysis, interactive decisions, mathematics discourse, mathematics student teachers, teaching decisions

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CHAPTER 1: INTRODUCTION

There is a national movement to reform how mathematics is taught in public schools (National Research Council, 2001; NCTM, 2000). This movement is trying to make reasoning and sense making the foundation of mathematics instruction (NCTM, 2000). Specifically, this reform is promoting a classroom discourse where students are expected to make and explain mathematical conjectures, and respond to other classmates' ideas (Sherin, 2002). Students are seen as the main source of mathematics in the classroom; the teacher becomes the facilitator of mathematical exploration and discussion (Sherin, 2002). B. E. Peterson and Leatham (2009) referred to the teacher's role in this classroom discourse as orchestrating a class discussion. The term *orchestrate* fits perfectly in this case because it requires a great amount of thinking and preparation on the part of the teacher to facilitate a good class discussion.

A large portion of orchestrating a good class discussion is using students' mathematical thinking (B. E. Peterson & Leatham, 2009). Using student thinking within whole class discussion is not trivial; in fact, it is quite difficult even for expert teachers (Ball, 1993; Sherin, 2002). The difficulty teachers face when trying to use their students' mathematical thinking has been noted in research (Franke & Kazemi, 2001; B. E. Peterson & Leatham, 2009; Stein, Engle, Smith, & Hughes, 2008). The fact that research has shown using student thinking to be difficult demonstrates our first problem: that we, as a field, do not fully understand how to help experienced teachers effectively use their students' thinking in a class discussion.

Even though effectively using student thinking is considered to be difficult, student thinking is highly valued within the field of mathematics education (Stein, et al., 2008; NCTM, 2000). One of the major difficulties teachers face with using student thinking in whole class discussion is the fact that decisions must be made in real time. This is our second problem: at most teachers have a couple of seconds to decide how to use student thinking. Making good teaching decisions in the moment is difficult.

Thus far I have brought up two problems with respect to an effective classroom discourse. The first is that we do not fully understand how to help teachers use their students' mathematical thinking. The second is that using student mathematical thinking requires the teacher to make in-the-moment decisions, which only adds to the complexity of using student thinking. Both of these problems are exacerbated for Student Teachers (STs), who are early in the process of learning to teach. First, STs are generally extremely new to teaching so they still have many aspects of teaching to work out for themselves. Hence, our limitations with helping experienced teachers use student mathematical thinking is even more limited due to the fact that STs have more than just using student mathematical thinking to improve. Second, using student mathematical thinking requires difficult in-the-moment decisions. STs are generally ill prepared to use students' mathematical thinking and yet make these decisions to do so in a matter of seconds. Still it is expected of STs to come into the field of mathematics education with the ability and knowledge to effectively orchestrate classroom discussions.

The focus of this study is to understand how STs' in-the-moment decisions with respect to using student mathematical thinking affect students' opportunities to make mathematical connections and to reason deeply about mathematics. By studying in-the-moment decisions we can better understand how STs use student mathematical thinking, and what makes this use difficult. Thus, we can take a step in the direction of better knowing how to teach STs to effectively use student mathematical thinking in order to better orchestrate class discussions.

I am particularly interested in this research because the most difficult aspect of teaching for me personally through my student teaching and now teaching experience is to orchestrate a productive mathematical discussion. Hence, if I am struggling with using student mathematical thinking then there are likely other novice teachers who also struggle with it. Thus, I hope to improve not only the preparation of STs; I also hope to improve my personal use of student mathematical thinking in my own classroom.

CHAPTER 2: THEORETICAL FRAMEWORK

STs make in-the-moment decisions with respect to orchestrating mathematical discussions, often as they try to use their students' mathematical thinking. These decisions influence how well the student thinking is used by the ST and, consequently, the students' opportunities to reason and learn about mathematics. The end results of these decisions are easily seen in how the STs used the student thinking. However, there are reasons behind the STs' decisions. Little is currently know as to what these reasons are or what they consist of. Yet, I submit that STs' reasons behind these in-the-moment decisions influence the students' opportunities to reason and learn about mathematics. This argument leads to my four research questions: (1) What decisions do STs make with respect to using their students' mathematical thinking? (2) Based on these decisions, how well do STs use their students' mathematical thinking? (3) What reasons are behind STs' decisions with respect to using student thinking? (4) How do the STs' reasons for the use of student thinking influence students' opportunity to learn?

In this chapter I outline a theoretical framework for exploring STs' in-the-moment decisions with respect to student mathematics thinking. To frame my first research question requires that I describe four phenomena: (1) I discuss my view of interactive teaching decisions and teaching routines; (2) I define the term *student thinking*; (3) I define the *use of student thinking*; (4) I discuss a broad framework that I used as a lens to view the use of student thinking. To frame my second research question, I give my view of teaching and learning. This view helps define the phrases *effective use of student thinking, ineffective use of student thinking,* and *students' opportunities to reason and learn about mathematics.* My view of teaching and learning leads to focused framework that I used to view the STs' classroom discourse with respect to students' opportunities to reason and learn about mathematics. To frame my third research question I describe my view of the reasons behind STs' decisions with respect to using student thinking. My fourth research question is related to the other three. Hence, no additional framework is needed to adequately describe it.

Interactive Teaching Decisions

As I am studying the in-the-moment decisions STs make with respect to using their students' mathematical thinking I must describe what I mean by a teacher *making an interactive decision.* My framework for interactive teaching decisions comprises the rest of this section.

While in the process of teaching a teacher is required to make many decisions in a brief amount of time (e.g., when to take roll, whether to listen Bob's sob story about how he lost his homework while three other students are trying to tell me something, whether to pass out the school newspaper right now, how Suzie's comment fits in today's lesson, whether I should have her better explain her reasoning). For this research I was only looking at a subset of decisions mathematics STs make in a given lesson. I needed a way to understand how STs make decisions and how cognitively demanding they are. Hence, I will define an interactive decision and describe how it helped this research.

A main focus of this research is to better understand how STs make in-the-moment decisions while teaching. These types of decisions are generally called interactive decisions (Clark & Peterson, 1986). Interactive decisions are decisions made in real time or decisions made in the moment. One major difficulty for all interactive decisions is that they must be made on the spot, which only adds to the complexity of the decision making process. Part of my focus is the interactive decisions STs make with respect to using their students' mathematical thinking. Hence, the most useful and simplistic definition for interactive decision for this research is borrowed from Clark and Peterson (1986), who defined an interactive decision as “a deliberate choice to implement a specific action” (p. 274).

Over the course of a lesson a teacher has been shown to make as many as 36 interactive decisions (Marland, 1979). However, my unit of analysis is smaller than Marland’s. I define my unit of analysis and an interactive decision to be every time the ST responds to any form of student thinking. Marland defines an interactive decision as the teacher deciding to have a class discussion on a particular topic. Having a class discussion on a particular topic is what I refer to as an episode of student thinking, which is a discussion focused on one particular mathematical idea and may have many comments and questions back and forth between the students and teacher. Marland attributed an episode of student thinking to one interactive decision, while I consider an episode of student thinking to have multiple interactive decisions. Thus, from Marland’s perspective, following through with the initial decision was still based on that one interactive decision. With my unit of analysis being smaller than Marland’s I recorded many more times the number of interactive decisions per lesson.

Given the large number of interactive decisions made by a ST over the course of a lesson it is necessary to narrow the focus of interactive decisions within this research. Hence, this research is focused only on interactive decisions made by the ST with respect to student mathematical thinking. This focus also includes how the ST reacts to student mathematical comments, gestures or written work. Even though I was only interested in a portion of interactive decisions, I found a substantial number of these types of decisions in the lessons I observed. This focus on ST interactive decisions with respect to student thinking allowed me to better isolate the STs’ reasons behind their interactive decisions.

Teaching Routines

Teaching routines are important to this research because I needed a way to describe how and why a ST would consistently react to student thinking in a similar way. Teaching routines are key to this research because teachers develop particular ways to respond to similar situations (Borko & Shavelson, 1990; Clark & Peterson, 1986). Thus, teaching routines require fewer cognitive resources for the STs to make a decision. While collecting data I looked for teaching routines; I looked for the STs to react in particular ways when placed in similar situations. Looking for teaching routines helped me to focus on why the STs used student thinking in a particular manner.

Clark and Peterson (1986) and Borko and Shavelson (1990) have both agreed that teaching routines play a major part behind teacher (and therefore ST) interactive decisions. They assert that teacher decisions become routinized, meaning that when the teacher receives student thinking the teacher simply decides to respond in a way that has previously brought about beneficial results. Teaching routines are useful for teachers because it allows teachers to use student thinking in ways that are perceived as beneficial quickly, easily and without a large amount of thought (Borko & Shavelson, 1990; Clark & Peterson, 1986). The following paragraph is an example of how a routine could be formed by a ST.

When a ST is put into a new teaching situation, for which they do not have a teaching routine, the ST must decide how to respond. An example of a ST being placed into a new teaching situation could be when a student talks about mathematics in a way that the ST has never thought of before. Hence the ST must make an interactive decision in how to use (or not use) this student's mathematical thinking. Because the ST is in unfamiliar territory they have to consider possible reactions. Some of the possibilities considered by the ST could consist of the following actions: the ST could brush off this student's comment; the ST could have the student explain his reasoning a little more; or the ST could just call on someone else to explain what the first student was thinking. After having made their choice, in real time, the ST carries out the action and the situation will play itself out. After the fact the ST will decide if the outcome of the interactive decision was beneficial. If the outcome was beneficial then the ST might react in a similar way the next time a similar situation arose, and this kind of reaction may eventually become a routine. If the outcome was seen as needing improvement then the next time a similar situation arose the ST might react in a somewhat similar way while trying to change a few things. Once the outcome is seen as being positive then the ST will make it into a routine, and consistently respond in the perceived best way.

Teaching routines are not inherently good or bad, but each routine may affect students' learning differently. A specific decision made by the teacher could facilitate students' learning but it might also encourage a louder classroom. Whether this decision is incorporated into a routine or not depends on the goals and beliefs of the teacher. If the teacher's primary goal is to facilitate students' mathematical learning with a secondary goal of classroom management they would probably incorporate the above decision into a routine. This is because their primary goal was met, where a secondary goal was not. However, if a different teacher's primary goal was to maintain control of a quiet classroom with a secondary goal of having students share their thinking then they probably would not incorporate the above decision into a routine because a quiet classroom was not maintained.

Given that STs are learning to teach we can infer that they are developing and adding to their set of teaching routines. Because of the newness of STs they are often put into teaching situations where they have not developed a routine. When this happens research has shown that

STs tend to be inflexible with student thinking and they tend to respond by "sticking to the plan" or their lesson outline by ignoring or not using the student thinking (Byra & Sherman, 1993). By regularly ignoring student thinking, two possible outcomes could occur. The first is STs could convey the message that student thinking is not wanted or needed within the classroom. The second is student confusion might not be resolved. For most teachers both of these are seen as negative outcomes. Hence, student teaching programs try to ensure that STs develop routines that produce valuable discourse patterns within the classroom. These discourse patterns should effectively use student thinking during class discussion.

Defining Student Thinking

As I am studying the decisions STs make with respect to using their students' mathematical thinking I must explain how I am viewing student mathematical thinking (student thinking). Student thinking is significant to this research because it is an idea upon which the rest of my study is built. What student thinking means exactly is therefore important to define because without it, inaccurate views of my research could easily result. Thus, I will define student thinking.

Without a context or definition the term *student thinking* could be interpreted to have several different meanings, therefore to be clear I must define what I mean by student thinking. For this research I am defining student thinking to be a student's verbal comment, body language or written work that has mathematical content and that is shared with the class. Student thinking that has mathematical content gives the teacher and the rest of the class an opportunity to make sense of a particular student's thinking. The teacher could then use this student thinking to help the students to move beyond their current understandings.

This definition of student thinking helped me keep my focus during data collection. I knew while collecting data that different forms of student thinking would arise. So I needed to know what to look for (e.g., a student question, comment, a response to a question, written work on the board). The definition of student thinking helped me focus my attention on student thinking and how the STs used it.

Using Student Thinking

Having defined *student thinking*, I now explain how I view the *use* of student thinking. Within this research I am defining the phrase *using student thinking* to be the teacher eliciting student thinking, making sense of the thinking, deciding (an interactive decision) how to respond (often with a routine) to the student thinking and then implementing the decision made.

Student thinking is a form of communication, therefore it does not represent a perfect picture of what the student is actually thinking (von Glasersfeld, 1983). Given that the interpretation of student thinking is problematic, its use is also problematic. However, it is still the best source of information as to what the students' current mathematical conceptions are. Thus, teachers need to be able to use student thinking in a way that will help their students develop robust mathematical conceptions. Within this research I wanted to see how STs would use student thinking given that its use is problematic. Effectively using student thinking is not trivial; in fact it is quite difficult even for expert teachers (Ball, 1993). In order to provide the opportunity for students to reason and learn about mathematics, teachers must make sense of their students' mathematics. The main method available to teachers to do this is through consistent eliciting of student thinking. To properly use this method teachers must elicit and use student thinking in a way that the teacher can find out how the student has constructed a mathematical concept and then push on this understanding. By doing so, the student will see the need to change their current thinking to something more compatible with their teacher's or entire class's. This method is generally compatible with a constructivist perspective of learning (Confrey, 1990). A teacher elicits student thinking by having a student verbalize their thinking or show their thinking in written form (writing their solutions on the board). To be able to elicit student thinking it is easier if the teacher has created a classroom environment where sharing one's thinking is accepted and generally practiced. Often elicited student thinking must be clarified, so the teacher may ask clarifying questions as part of the elicitation process. After student thinking has been elicited the teacher will try to make sense of the student's mathematics and then decide how to use that thinking.

Given my definition of using student thinking it is possible for a teacher to "use" student thinking by making sense of the thinking and then deciding not to address it. Or, a teacher could use student thinking by making sense of the thinking and then deciding to talk about something the student said. Hence, there are different ways to use student thinking. So, I needed a way to describe the STs' response to the student thinking, or how the STs used the student thinking with these routines. This is why I developed a broad lens to categorize the "uses" of student thinking. The categories are Do Not Use, Teacher Talk, and Run With. Do Not Use is easily seen in a lesson. Do Not Use occurs when the ST receives the student thinking and chooses to do nothing with it. The ST gives no response to the student thinking. The second, Teacher Talk, occurs when the ST receives the student thinking and decides that the student thinking contains something they want to talk about. So, the ST talks about the student thinking. The third, Run With, describes a use of student thinking, in that the ST receives the student thinking and then decides that it would be good to have a class discussion about that particular topic. This broad analysis of the use of student thinking helped me to better describe how the STs used student thinking.

My View of Learning and Teaching

As I am studying how well STs use their students' mathematical thinking I must describe my view of learning and teaching. My view of learning and teaching is integral to how I define the phrases *effective use of student thinking, ineffective use of student thinking,* and *students' opportunities to reason and learn about mathematics.* It is also from my view of learning and teaching that I built a discourse analysis to describe how well the STs used student thinking. In this section I give my view of learning and teaching, then I give a few definitions of phrases I used. I conclude with a brief description of how my view of teaching and learning influenced my focused discourse analysis with respect to using to student thinking.

For this research I am using constructivist learning theory. By saying this I mean that individuals actively construct their own understanding rather than absorb or assimilate the understandings of others (Simon & Schifter, 1991). Simon and Schifter (1991) explained that learners construct new understanding when they are placed into a problem situation which disturbs the learners' current organization of knowledge; this perturbation occurs because the learners' current understanding does not adequately characterize, solve or explain their current situation. Hence, this perturbation causes learners to reorganize their current conceptual structures to take into account the problem situation.

von Glasersfeld (1983) explained that written or spoken words are not containers by which the writer/speaker conveys meaning. Instead everyone has to abstract what each word means from their own experiences with that particular word. Thus the meaning of a word is subjective to every person's experiences and understanding. This poses a problem: if all of our words are subjective, then communication must be extremely difficult if not impossible, yet we all communicate on a daily basis. von Glasersfeld explained that we are able to communicate without relative difficulty as long as each communicator's representations are compatible with the other's such that what is being conveyed does not conflict with "the situational context or the speaker's expectations" (p. 68).

Viewing learning with a constructivist perspective has several implications for teaching. One implication for teaching is that transmission of knowledge is not effective. Confrey (1990) suggested that "when one applies constructivism to the issue of teaching, one must reject the assumption that one can simply pass on information to a set of learners and expect that understanding will result" (p. 109). Communication is a complex process, and because of its complexity we must come to understand its failings. The biggest shortcoming is that we cannot accurately share knowledge.

An example of this inaccuracy is easily seen when students incorrectly extrapolate what the teacher is trying to teach. To illustrate this point consider the following vignette. In my Business Calculus recitation my students had just been to a lecture on anti-differentiation. Within this lecture they covered several rules of how to take an anti-derivative. The rule that was most often used in the first section of homework is what I refer to as the backwards power rule. This rule basically states that you can undo a derivative of the terms of a polynomial expression by adding 1 to the exponent then dividing by the newly formed exponent. This rule generally works except for $\frac{1}{x}$. In this case the anti-derivative is the function $\ln(x)$. However, many students try to misapply the backwards power rule by performing a procedure similar to the following $\int \frac{1}{x} dx = \int x^{-1} dx = \frac{x^0}{0} + c$. I have seen this difficulty before, so I tried to make it very clear that the preceding mathematical argument was incorrect. By so doing I unknowingly confused many of my students into thinking that $\int \frac{1}{x^2}dx = \int x^{-2}dx = \frac{x^{-1}}{-1} + c$ was also an incorrect solution method. My students' misconception, that I unknowingly encouraged, was brought to my attention when I graded a quiz my students took later that day. This is a good example of how my attempt at transferring my knowledge did not result in achieving my learning goals.

If a transmission style of instruction is inadequate, then what type of teaching should take place? Confrey (1990), after having presented a similar argument, suggested the following alternative:

> When teaching concepts, as a form of communication, the teacher must form an adequate model of the students' ways of viewing an idea and s/he then must assist the student in restructuring those views to be more adequate from the students' and from the teacher's perspective (p. 109).

With my Business Calculus students I made no, or very little, attempt to understand their way of viewing integration, which made it impossible for me, as the teacher, to help my students change their views to a more adequate form. Had I relied on constructivism to better guide my teaching, I feel that my students' learning outcomes would have been more positive.

Now that I've explained my view of learning and how my views of learning have implications for teaching, I need to explicate my view of teaching. My view of teaching is important to my research because I wanted to build a discourse analysis that not only focused on the students sharing mathematical thinking but also I wanted to focus on the ST to see how well they used to their student's thinking. The discourse analysis that I developed helped me to focus on how the students and STs were participating in the classroom discourse. My view of teaching played a major role in how I viewed STs' use of student thinking.

I view teaching as facilitating student learning. This perspective of teaching is different from many perspectives that might view teaching as "imparting knowledge or skills, or to give instruction" (from dictionary.com). I purposely avoided these two definitions of teaching because they are both problematic. First, thinking of teaching as imparting knowledge or skills implies that knowledge or skills can be shared in an unproblematic way between individuals. From a constructivist point of view this is not the case (von Glasersfeld, 1983). Learning is a problematic process. Second, viewing teaching as giving instruction is problematic because if a teacher were to teach by strictly giving instruction, or transmitting knowledge, the teacher does not know if their students are learning. What is the point of teaching if students are not learning? This is why I view teaching as facilitating student learning. This view is also related to Simon and Schifter's (1991) view of teaching mathematics: "Teaching mathematics is to be understood as providing students with the opportunity and the stimulation to construct powerful mathematical ideas for themselves and to come to know their own power as mathematics thinkers and learners" (p. 310). Viewing teaching as the facilitation of learning better lends itself to the complexities of teaching where the simple transmission view of teaching is insufficient.

Because of my view of teaching and learning, I evaluate the quality of STs' discourse by evaluating the extent to which they effectively use student thinking. There are many different ways to use student thinking. There are effective ways to use student thinking and ineffective ways to use it. My view of teaching and learning helps classify the effectiveness of different uses of student thinking. Hence, I need to define and clarify the phrase *effective use of student thinking*. This phrase means that within an episode of student thinking the teacher used student thinking in a way that promoted students to reason and make sense of mathematics. An *ineffective use of student thinking* occurs when the teacher uses student thinking in a way that discouraged or impeded students from reasoning or making sense of mathematics. Ineffective uses of student thinking can promote students to guess. A portion of ineffective uses contain some appearances of effective uses. With these uses the ST may repeat a lot of student thinking but not promote students to reason, or the ST may initiate a good discussion but not be able to capitalize on it. Upon close inspection these uses of student thinking are still ineffective. I refer to these types of ineffective uses of student thinking as a marginally ineffective use of student thinking or a marginal use of student thinking.

My view of learning was also helpful to identify classroom discourses that gave students an opportunity to reason and learn about mathematics, or what I am calling an *opportunity to learn*. What I mean by the phrase *opportunity to learn* is that the ST seemed to view the sharing of knowledge as problematic and the STs effectively used their students' thinking. Another way to look at opportunity to learn is discourse patterns where students were placed into situations in which they had to think about and share their mathematical ideas with others. The discourse patterns that give students and opportunity to learn are different from traditional instruction because students would be expected to be an integral part of the mathematics lessons instead of being a bystander who watches the lessons.

Framework of Classroom Discourse

My view of teaching and learning allowed me not only to describe the teaching that took place but also to see the extent to which it facilitated students' opportunity to learn by creating a classroom discourse that promotes students to make mathematical connections and to reason deeply about mathematics. Were the students' mathematical conceptions ever perturbed in order to create a robust understanding? Or did the ST rely on lecturing to convey knowledge? Based on my view of learning and teaching I developed a framework to describe how the STs helped students clarify their thinking, how the students participated in class discussions, and how well student thinking was used. A description of this framework follows.

I needed a framework that not only describes what the ST was doing in the classroom but also what the students were doing. It is important to look at how the students are contributing to the discourse because with a reform oriented discourse or class discussions the students play a major role. They can no longer be considered passive listeners. So, my framework of classroom discourse is broken into three parts. The first part, Better Understand (BU), is focused on what the ST is doing. Is the ST eliciting student thinking? Is the ST asking clarifying questions? Is the ST creating opportunities for herself and her students to make sense of student thinking? Is the ST trying to better understand or construct a more complete model of their students' current mathematical understanding, or not? Hence, the Better Understand portion on my discourse framework is focused on the ST's classroom discourse. BU is part of what makes up an effective use of student thinking, consequently, BU is also partially how I identified if student thinking was well used.

The second portion of my framework, Evidence of Student Mathematics (ESM), is focused on how the students are participating in the classroom discourse. Are the students showing evidence that they are thinking by bringing new mathematical ideas to the lesson? Are the students building on or extending other's mathematical thinking? Or, are the students merely listening to and responding to the ST's questions? ESM is important to my classroom discourse analysis because the students are important to the classroom discourse. ESM gives evidence that student thinking is being effectively used. Hence, ESM is also partially how I identified if student thinking was well used.

The third and final portion of my classroom discourse framework is Well Used (WU). WU is an overall classroom discourse framework that determines how well the student thinking was used. Did the overall class discussion give students the opportunity to learn by deepening their understanding of mathematics? This discourse framework ties together the teacher's actions (BU) and the students' actions (ESM) within the whole classroom discourse. WU is essentially a measure of the effectiveness of the use of student thinking with the use of BU and ESM. Thus, my view of learning and teaching helped me build a framework of the classroom discourse I observed in this study.

Reasons Behind Decisions

As I am studying the reasons behind STs' decisions with respect to using student thinking I must define and describe reasons that are used to make decisions with respect to student thinking. There are actually two significant definitions of a reason for this research. The first definition of a reason is "a basis or cause, as for some belief, action, fact, event, etc" (from Dictionary.com). Hence, in the ST's mind when they are making a decision they use these types of reasons as a basis for making their decisions. The second definition of a reason is "a statement presented in justification or explanation of a belief or action" (from Dictionary.com). When I collected data I asked the STs for the reason (a verbal justification of an action) why they used student thinking in a particular manner. Hence, I successfully collected and recorded the second type of reason. Unfortunately, what I really need for this research is the first type of reason. However, as I am subscribing to constructivism this type of reason is unavailable to me as the researcher because I am unable to look into the ST's mind to see what is going on. Hence, I am resigned in assuming that the verbal response reason is the same as the reasons that the STs based their decisions on. For the most part this assumption is probably true, but there is the possibility that misalignment occurred.

Another important aspect of reasons comes from the routines aspect of this framework. When the ST is initially creating a teaching routine they rely on reasons to make the first decision to act a certain way. However, as this decision process becomes a routine the reasons behind the routines become less important because the ST is no longer considering the reasons as part of the teaching decisions. They simply see an action as needed so they perform the needed action. A teaching routine may change over the course of time and be adapted into different uses without being reassessed or compared against the original reasons that were used to create it. Hence it is possible to have conflicting reasons with routines because the STs have stopped thinking about the reasons behind the routines.

CHAPTER 3: LITERATURE REVIEW

To better situate my research it is important to note what other research has been done involving interactive teaching decisions and the use of student thinking. Interactive teaching decisions have been the topic of research studies for some time. These studies have used information processing as their theory of choice (Borko & Shavelson, 1990; Byra & Sherman, 1993; Clark & Peterson, 1986; Marland, 1977; Westerman, 1991). Because of an information processing theory, all of these studies have tried to create models and schemas that map out how interactive teaching decisions are made. However, for my research, I am using a constructivist learning theory. This difference in theories makes aspects of previous research unusable and unneeded for my research (i.e., creating models or computer programs to describe how interactive teaching decisions are made). Nevertheless, many of the findings of this previous research are important and significant (e.g., the differences in decisions made between expert teachers and novice teachers). The pertinent findings from these interactive teaching decisions studies are discussed later in this chapter. However, it is important to note that my research was different than the previous research because of my use of constructivism as a learning theory.

The use of student thinking has been a more recent topic of study (Confrey, 1990; Doerr, 2006; Fraivillig, Murphy, & Fuson, 1999; B. E. Peterson & Leatham, 2009). This more recent interest in using student thinking in whole class discussion is related to a shift in the teaching of mathematics to be "based on reasoning and sense making" (NCTM, 2000, p. 1). Effectively using student thinking is tied to this shift in teaching because, as B. E. Peterson and Leatham (2009) said, "the purpose of such use is to help all students to gain a better understanding of the concept at hand" (p. 102). The way in which my research differed from previous research on the use of student thinking is that I do not only want to see how STs are using student thinking. I also want to find out what reasons they give for using student thinking in a given way. This means that my analysis went a step deeper in that I analyzed the decisions STs make with respect to using student thinking and how well they use it. I analyzed the reasons the STs gave for their actions. I also tied the uses of student thinking to the reasons why the STs acted the way they did. Some uses of student thinking will be discussed later in this chapter.

This literature review is divided into three main sections, all of which describe literature related to my research questions: 1) What decisions do STs make with respect to using their students' mathematical thinking? 2) Based on these decisions, how well do STs use this thinking? and 3) What reasons do STs give for their use of student thinking? My fourth research question tries to draw connections between my first three research questions, and I found no such research attempting to make these connections. Hence, the first section of this literature review covers research on how teachers or STs make interactive decisions. The second section covers effective and ineffective uses of student thinking. Effective and ineffective uses of student thinking give insight as to how well student thinking was used. The third section discusses reasons behind uses of student thinking.

Teacher Decision Making

The topic of ST and expert teacher interactive decisions has been studied regularly. For example Westerman (1991) studied the thinking and decision making of STs and expert teachers (the STs' cooperating teachers). The expert teachers thought about their teaching from the students' perspective and tended to adapt the lesson due to student thinking. However, the STs tended to stay with the lesson plan and not adapt to student thinking. Byra and Sherman (1993) also noted that STs tended to be inflexible with their use of student thinking. Westerman's (1991) and Byra and Sherman's (1993) research is significant because it describes how STs tend not to effectively use student thinking, especially when it was unanticipated. My research builds on this work as it focuses on trying to understand the reasons why STs tend to use student thinking in unproductive ways. Thus, by understanding STs' reasons for using student thinking in the moment of an interactive decision we better understand how student teachers perceived the situation, which lends insight to as to why the STs used student thinking they way they did.

Clark and Peterson (1986) reviewed twelve studies that involved teacher interactive thoughts and decisions. More specifically six of these studies involved the content of teacher interactive thoughts. Clark and Peterson explained that most often expert teachers' interactive thoughts were concerned with the learner. Surprisingly a relatively small amount of teachers' interactive thoughts were concerned with instructional goals or lesson objectives. These expert teachers have learned to focus their instructional attention on the students' perspectives and not the lesson goals. Clark and Peterson addressed the interactive thoughts of expert teachers, or at least more expert than STs. Thus, it is important to understand the focus of STs' interactive thoughts. By understanding STs' in the moment justification of this thinking then we can better address how to overcome problematic types of thinking (i.e., the ST thinks that they are effectively using student thinking when in fact they are not) to improve STs' teaching methods.

Borko and Shavelson (1990) based their review of the research on teacher interactive decisions on Clark and Peterson's (1986) work. However, Borko and Shavelson (1990) were able to give a more recent review of the literature in which they pointed out that most of decisions teachers make when teaching are simply reactions to the class environment. An action is seen as needed, so the action is performed. This type of interactive decision has been referred to as a "deliberate act" (Marland, 1977) or an "elective action" (Shroyer, 1981). However, a very small portion of teacher interactive decisions are actually deciding between two or more alternatives (i.e. should I give another example or should the class start working in small groups). In this research Borko and Shavelson (1990) help show the difficulty in constructing a teacher decision model. This influenced my research by pushing me away from attempting to develop a complete teacher decision model with an information processing theory; instead, I have focused on understanding STs' justifications for their decisions and how they affect the classroom discourse.

Before moving on I must make a note that for the most part research on teacher interactive decision making occurred in the 1970s and 80s which creates a large gap in this literature review. Borko, Roberts and Shavelson (2008) accredited this gap to three main problems. First, these studies were unable to make the link between teacher decisions and student learning outcomes. Second, there was no link between teacher content knowledge and instruction type. Third, little that was worthwhile or remarkable emerged from these studies. Because of the development and progression of the field of mathematics education each of these three problems has been or can be resolved. First, we have repeatedly come to the conclusion that students will not learn what they have not been given the opportunity to learn (Hiebert, 2003). By viewing learning from a constructivist perspective we can see that if STs can effectively use student thinking in the very least they are giving their students the opportunity to learn. This successfully forms a link between teacher decisions and student learning outcomes. Second, teaching mathematics in the 1970s and 80s compared to teaching mathematics in a reform oriented manner today is a stark contrast. This is especially true to understanding different discourse types. In the 1970s and 80s the discourse pattern of most note was the I.R.E. cycle (Mehan, 1979). Nowadays classroom discourse is a large area of study in education. Hence, we can better understand the effectiveness of instruction types because we better understand different discourse patterns. Lastly, the fact that little remarkable or noteworthy results came from these studies has to do with their focus. The majority of studies from this time period focused on categorizing the different types of interactive decisions. All of these studies were missing teachers' justifications for their interactive decisions. I find teachers' justifications more meaningful than categorizing their decisions. Besides the reasoning presented above the three problems presented by Borko, Roberts, and Shavelson (2008) are all partly resolved by the fact that issues from the 1980s are no longer issues today because the field has grown and changed. Thus I can echo Borko, Roberts, and Shavelson by saying that because of "advances in theory and research findings…. teacher decision making may, once again, be a useful construct for the study of teaching linked to student outcomes" (p. 65) .

Using Student Thinking

To better situate my research and to better answer my research question of how STs use student thinking I must discuss how STs, or even teachers, have been shown to use student thinking. This section is divided into two parts: effective uses of student thinking and ineffective or problematic uses of student thinking.

Effective Uses of Student Thinking

As briefly discussed in the last chapter, there are several ways to effectively use student thinking. One of the least complex ways to effectively use student thinking is described by van Zee and Minstrell (1997) as a reflexive toss. A reflexive toss was defined as "utterances with which [the teacher] elicited further thinking by catching the meaning of the student's prior utterance and throwing responsibility for thinking back to the students" (p. 241). A reflexive toss illustrates an important part of effectively using student thinking: giving students the opportunity to reason for themselves. NCTM (2000) has placed a renewed focus on the importance of student sense making and mathematical reasoning.

Research has shown that teachers who effectively use student thinking often follow a very specific teaching cycle: The teacher elicits student thinking, makes sense of the thinking, decides (an interactive decision) how to push on the student thinking and then implements the decision made. The key difference between this effective use cycle and the use cycle discussed previously is that the effective use cycle has the inherent goal of pushing on student thinking to help students reason and make sense of mathematics where the generic cycle of use does not. Different versions of this complete effective use cycle and portions of it have been noted in research (Confrey, 1990; Doerr, 2006; Fraivillig, et al., 1999; B. E. Peterson & Leatham, 2009; van Zee & Minstrell, 1997). A more complete discussion of this cycle follows.

Eliciting student thinking is a significant part of using student thinking (Confrey, 1990; Fraivillig, et al., 1999; van Zee & Minstrell, 1997), but it is not the end goal. Instead, it is a necessary first step in effectively using student thinking. For example, even before Minstrell (van Zee & Minstrell, 1997) could use student thinking with a reflexive toss he had to first elicit student thinking. Confrey (1990) stated, in reference to using student thinking, that "teachers need to create as many and as varied ways of gathering evidence for judging the strength of a student's constructions as possible" (p. 112). One way to gather such evidence is through eliciting student thinking.

Making sense of the students' current mathematical conceptions is an integral part of effectively using student thinking (Doerr, 2006). Actually, the ST must make sense of the student thinking before they are able to use it (B. E. Peterson & Leatham, 2009). B. E. Peterson and Leatham asserted that listening to student thinking must be done with the intent of using it;

hence, the student thinking will not be used until the ST (or teacher) recognizes the student thinking as a ‘teachable moment.’

Pushing on student thinking, the final part of this cycle, is to use the student thinking to help the students develop a more robust understanding of mathematics. Wood (1998) defined a version of pushing on student thinking as focusing. Doerr (2006) noted that in order to push on student thinking students needed to be placed in appropriate situations based on their current mathematical conceptions. Fraivillig et al. (1999) reported that an expert teacher pushed the students to learn by supporting and extending the students’ thinking. Pushing students to learn can be done by challenging students to go beyond their initial solution methods, or having students try to find more efficient solution methods. Pushing students to learn can also be accomplished by the use of student generated problems. All of these are examples from research that demonstrate pushing on student thinking.

Ineffective Uses of Student Thinking

It might seem that ineffective uses of student thinking would be the worst possible discourse pattern that could occur in a mathematics classroom, but I do not believe this is so. Ineffective uses of student thinking are at least a step above the traditional telling style of mathematics instruction because the teacher (or ST) is trying to use student thinking and incorporate it into the lesson. However, it seems that many of these “uses” are just as problematic or unproductive as teaching through telling. Different forms of ineffective uses of student thinking have been called the I.R.E. cycle (Mehan, 1979), funneling (Wood, 1998), teacher lust (Tyminski, 2010), and naïve uses of student thinking (B. E. Peterson & Leatham, 2009). A brief description of each of these ineffective uses of student thinking follows.

The I.R.E. cycle occurs when the teacher asks a question, the student responds, then the teacher evaluates the student's response. The problem with this use of student thinking is that it requires little thought or understanding on the part of the student. Wood's (1998) funneling has the same basic problem as the I.R.E. cycle. This funneling discourse pattern occurs when the teacher guides a student through the solution to a problem and only asks students lower-order questions. Funneling then allows a student to be "led" through a problem, while answering questions correctly. However, there is little or no evidence that the student actually understands or could reproduce the mathematics at hand. Thus, funneling allows students to correctly answer questions without understanding them.

Teacher lust occurs when a teacher acts in a way that removes the opportunity for a student to think about or engage in the mathematics for themselves (Tyminski, 2010). This is often done by the teacher thinking about or engaging in the mathematics instead of the students. Again, this use of student thinking is problematic because the students are not required to engage in the mathematics. Teacher lust is related to the I.R.E. cycle and funneling in that all three of these discourse patterns remove the necessity for students to reason deeply about the mathematics, where as effective discourse patterns encourage students to reason deeply about the mathematics.

B. E. Peterson and Leatham (2009) describe naïve use of student thinking as when "the STs attempted to use their students' thinking but fell short" (p.114). Naïve use had three distinct forms. 1) Student thinking used as a trigger. This means that the ST listened to the student thinking, but then used a portion of the thinking to redirect the class discussion to a direction the ST wanted it to go. 2) The mere presence of the correct solution. This means that the ST used the student thinking enough to see a student's misconception then the ST thinks they cleared up the misconception by talking about or emphasizing the correct solution. 3) The mere presence of multiple solutions. This occurs when the ST thinks that by exposing the students to multiple solutions or solution strategies students will automatically learn or understand the underlying mathematics that connects the multiple solutions or strategies. Stein et al. (2008) called this naïve use of student thinking a show and tell discourse.

Reasons for Interactive Decisions

Reasons for interactive decisions were very scarce in the literature. However, Marland (1979) gives some insight into STs' reasons for interactive decisions. Marland studied six elementary school teachers' interactive decisions made within two lessons. Four of the twelve lessons where mathematics lessons while the remaining eight were language arts lessons. Marland noted that often the teachers would make estimates or guess as to what the students were thinking. Rarely would the teachers follow up to see if their estimates were correct, instead they would proceed as if their estimates were accurate. Also interview data showed that these teachers rarely thought about their lesson plan and that they did not reflect much on their own behavior and its affect on the lesson. These are some interesting results because it has been previously noted that teachers think little about their lesson plan while teaching (Clark & Peterson, 1986), but the results also shows that little reflection about teaching was common for these teachers. Marland's (1979) work briefly touches on STs' reasons behind interactive decisions. Marland saw STs' reasons behind interactive decisions when the STs gave reasons for or justified their actions. Evidence of these reasons can also be seen in the ST's actions because they proceeded as if their estimates were accurate. Through this research I hope to find and classify STs' reasons for their use of student thinking.

From the articles on teachers' interactive decisions or thoughts we saw that there are many varying reports as to what teachers' interactive decisions consist of. However, there is very little research on teachers' or STs' reasons as to why they chose to use student thinking in the chosen manner and how these reasons affect the opportunities given by STs for their students to learn. To better understand STs' rationale behind their teaching we need to understand not only the interactive decisions they make but also their reasons for such and how these decisions affect the classroom discourse.

Conclusion

The research reviewed above has primarily focused on either teaching decisions or the use of student thinking. My research built on these studies in that I sought to connect these two bodies of research. My research focused on the types of reasons STs give while making interactive teaching decisions, hence, my focus on teaching decisions. My research also focused on how STs used their students' mathematical thinking. However, the key aspect of this research focused on the STs' reasons given for using student thinking and how the reasons the STs gave for their interactive decisions influenced the students' opportunity to learn. These ideas are currently absent from the literature.

CHAPTER 4: METHODOLOGY

This chapter is broken up into three sections. The first section is a description of the data collection including the study's participants, tools that were used in data collection, and how data were collected. The second section includes a description of the discourse analysis and examples of how the discourse analysis was conducted. The third and final section is a description of the analysis of the STs' reasons for using student thinking.

Data Collection

This research consisted of case studies of two Brigham Young University (BYU) mathematics STs, Megan and Olivia (pseudonyms). Megan and Olivia were both seniors at BYU who were preparing to graduate with a Bachelors of Science in Mathematics Education. The only requirement left for Megan and Olivia to earn their degrees was student teaching. The selected STs participated in this study on a volunteer basis. The context of these STs' student teaching program is important to this study. Within their student teaching program each ST was given multiple opportunities to reflect on and develop the ability to use student thinking within a mathematical discussion. These BYU STs were also grouped into student teaching pairs—Megan and Olivia were paired together in one classroom with a single cooperating teacher. Each pair collaborated and planned together to create their lesson plans. They also reflected on each others' teaching. Having a second ST in the classroom helped the participants focus more on teaching and using student thinking while focusing less on classroom management.

The data for this study came from 7 lessons from each ST. The number of lessons was originally planned to be between 5 and 7 and, in order to saturate my data with different uses of student thinking, I ended with 7 lessons. The number of recorded lessons also reflects the amount of data collection that seems appropriate for a master's thesis. The type of lesson that was most beneficial for this research was a lesson where new mathematical material was being covered and classroom discussion was taking place. This lesson selection was done with the intent to better saturate my data with different uses of student thinking. Hence, I tried to avoid review lessons or testing days. However, as I collected data at my participants' convenience, I did end up collecting data during two review days.

For each lesson I attempted to collect (a) a short pre-lesson survey (for 11 of the 14 lessons), (2) a copy of any written lesson plan (for 5 of the 14 lessons) and any other lesson materials (for 4 of the 14 lessons), (3) a video recorded class lesson (for all 14 lessons), (4) field notes (for all 14 lessons), and (5) a video recorded post-lesson interview (for all 14 lessons). I also collected a final video-recorded interview with each ST. All data connected to a given lesson were collected on the same day as the lesson.

The pre-lesson survey (see Appendix A) was given to the STs on a printed piece of paper prior to filming. STs filled out the brief pre-lesson survey informing me of their lesson/unit goals, and of any anticipated student thinking or questions that they might want to incorporate into the lesson. Unfortunately the STs were very busy and often neglected to fill out the pre-lesson survey; this neglect also occurred with the creation of written lesson plans. However, when this information was available, it was kept by the researcher and used with the prefabricated interview protocol form (see appendix B). Such interview protocols have been helpful for others to organize the data collection process (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000).

The video taken of each class lesson focused on the ST during whole class discussion. Thus, when filming and during analysis, student comments were included but individual students were not explicitly followed. Teacher conversations with groups were not included as part of my analysis. However, if student thinking had been elicited from a small group and been brought to the classes' attention then this episode of student thinking would have been included in my data and analysis. (No such instances occurred.)

The field notes helped capture anything the camera may have missed. Field notes are widely used and recommended (Borko, et al., 1992; Jick, 1979; Patton, 2002). The notes were taken on a prepared form (see Appendix C). The unit of analysis for this research is a ST response to student thinking. Student thinking is revealed within a mathematics lesson when a student makes a solicited or unsolicited mathematical remark with regard to the lesson. On this form I recorded and coded all episodes of student thinking. Besides coding episodes of student thinking (when the class discussion is focused on one idea for a period of time), I also noted the time the episode occurred and then calculated the time elapsed from the beginning of the lesson. This facilitated the post-lesson interview and analysis. I phrased post-lesson interview questions on the field note form.

Before the video recorded post-lesson interview I used my field notes from the lesson to determine which episodes of student thinking to discuss with the STs. I tried to discuss at least 5 episodes of student thinking per student teacher per lesson. These interviews lasted from 5-30 minutes, with Megan averaging around 10 minutes per interview and Olivia averaging around 20 minutes per interview. My intention in selecting the episodes of student thinking was to saturate my data with varied uses of student thinking (i.e., Do Not Use, Teacher Talk, and Run With). The types of episodes of student thinking I discussed with the STs changed over the course of data collection. While collecting data I quickly saturated "Teacher Talk" uses of student thinking. Hence, I stopped discussing "Teacher Talk" episodes and focused on discussing "Do Not Use" and "Run With" Episodes of student thinking. As these episodes occurred much less often, it took the remainder of the lessons to saturate these uses of student thinking. Once with Megan and once with Olivia I was limited by the ST not understanding or remembering one of the episodes of student thinking I described. I put off discussing these episodes until the final interview. In the video recorded final interview I reviewed and discussed the episodes of student thinking from the interviews which the STs could not sufficiently recall, and then found another episode to discuss with the STs that I had accidently missed in the original post-lesson interviews (see Appendix D). With this interview I used the actual video clip to stimulate the ST's memory as to what they were doing, as a few weeks had transpired between the observed lessons and the final interviews. Many other researchers have found stimulated recall a very effective tool for interviews (Clarke, 2001; Lyle, 2003; P. L. Peterson & Clark, 1978; Shimizu, 2002, April). I also found stimulated recall to be an effective tool for this research. However, I chose not to use stimulated recall for the post-lesson interviews due to how quickly I was able to interview the STs after they taught the lesson (no more than two hours had passed since the lesson was taught.) Thus the STs had very little difficulty remembering what had transpired and the reasons why they used their students' thinking in a particular manner (see Figure 1).

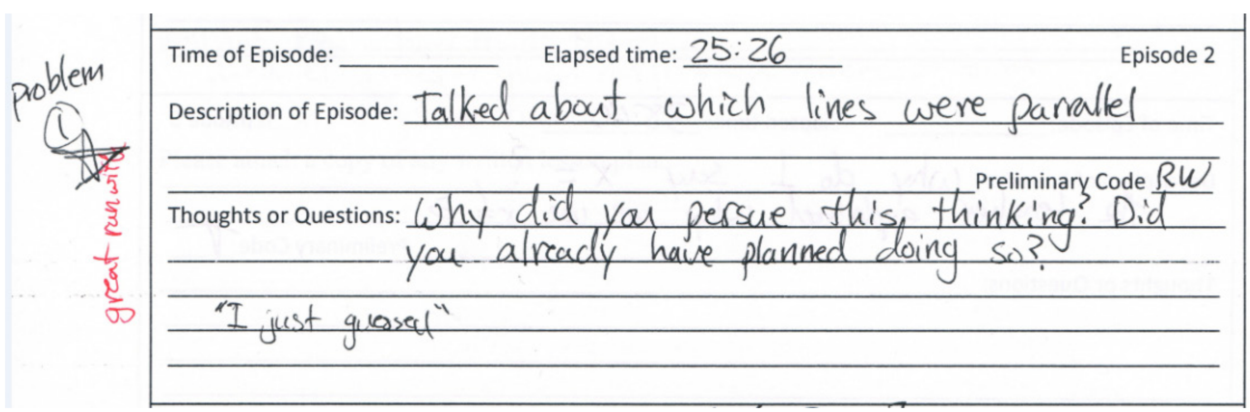

problem ①
great run with
Time of Episode: ____ Elapsed time: 25:26 Episode 2
Description of Episode: Talked about which lines were parallel
Preliminary Code RW
Thoughts or Questions: Why did you persue this thinking? Did you already have planned doing so?
"I just guessed"

Figure 1. An example of an episode of student thinking recorded in my field notes.

Methodologies similar to mine have been used many times, (e.g., Clarkson, 2000; Golombek, 1998; Steffe & Thompson, 2000; Westerman 1991). Clarkson's (2000) methodology was "to work with individual teachers using a cycle of a preliminary interview, a classroom observation, and a post-observation debriefing interview on the same day" (para. 2). Westerman (1991) also had a very similar methodology structure, described as having "audiotaped planning interviews, videotapes of lessons, stimulated recall interviews, post-teaching interviews, delayed self-reports" (p. 292). Within Westerman's methodology a failsafe was created so that if anything was not sufficiently clear after the post-teaching interview then the researcher would perform a second post-teaching interview shortly thereafter, which would take place via email. I purposefully collected data through different sources (collecting lesson plans, video recording lessons, and interviewing subjects) and at different times (prelesson, lesson, and postlesson) so that all of my conclusions could be verified through triangulation (Jick, 1979; Patton, 2002) with my final interview acting as a failsafe so I could follow up with anything that I felt unsure about. For example, through the course of interviewing the STs I felt that both STs believed that their students, in the classes I observed, could not reason deeply about mathematics. So, in the final interview protocol, I built in a question to verify this assumption.

Data Analysis of Observed Discourse

Here I describe in detail the three different aspects of my classroom discourse analysis that were briefly mentioned in my theoretical framework. The first is an analysis of Better Understand (BU). The second is an analysis of Evidence of Student Mathematics (ESM). The third is an analysis of Well Used (WU). These three aspects of classroom discourse made up my analysis.

Better Understand (BU)

BU is defined as the ST creating opportunities for both her and her students to make sense of student thinking. This can be done if the ST elicits student thinking directly (i.e. asking a clarifying question) or indirectly (i.e. creating classroom norms or situations where students feel the need to share their thinking). *This focuses on what the ST is doing or has done.* BU is not a single elicitation of student thinking; instead it must be at least two elicitations of student thinking on the same line of thought. So, in other words, a BU is an instance where the ST makes a move that helps her and her class to better understand what the student is saying. Please note the following example of a BU discourse pattern and when it becomes a BU discourse pattern.

Megan:	So let's do the fraction first. How do we get rid of the fraction?
Jim:	Times by the denominator.
Sue:	Multiply by negative three. (Student thinking was elicited, but this is not yet a BU)
Megan:	Why did you pick negative three? *(The ST asks a clarifying question)*
Sue:	Because it's negative. *(Student thinking was again elicited and now this episode is considered a BU) 1*
Megan:	Because what's negative? *(Another clarifying question)*
Sue:	The fraction. *(Student thinking was again elicited) 2*
Megan:	Because the fraction is negative. Okay.
Sue:	Oh, wait never mind. Don't times by a negative three. *(This is an example of indirectly eliciting student thinking. This is due to the fact that the ST did not necessarily ask for this input but the student felt the need to give it anyways. Thus the situation within the classroom that the ST has created elicited this student's thinking.) 3*
Megan:	Why not?
Sue:	Because then that way you can add it to the other side. *(Student thinking was again elicited) 4*
Megan:	So you saw this right now when you said I don't want that number in front of x to be negative, and so we could get rid of it right now. But we're going to add this to the other side. So we want this to stay negative so when we add it will be positive. Does that make sense?

BU is rated on the number of times student thinking was elicited past the original statement. As you can see above every follow-up elicitation of student thinking was numbered 1, 2, 3 and 4. Hence, this is an example of a BU-4. However, if student thinking is never elicited past an original statement in an episode (i.e. the IRE Cycle) then it is a BU of level 0. Within this research I have several examples of BU levels 0 through 2 whereas levels 3 through 5 rarely occurred.

The cycle of using student thinking as briefly discussed in the framework has four parts: first, the ST elicits student thinking, second they make sense of the thinking, third they decide how to address the student thinking, and fourth they implement the decision. BU fits in this cycle in two parts—first and primarily in eliciting student thinking, second and partially in making sense of the students' mathematics. BU is eliciting student thinking past an initial comment. BU then creates the opportunity for the ST and her students to make sense of a student's mathematics. I obviously was unable to see if the ST or students actually did make sense of the student thinking. However, with a BU there is at least an opportunity for them to do so. BUs are part of effectively using student thinking because they are needed to give students the opportunity to make sense of mathematics.

Evidence of Student Mathematics (ESM)

ESM is defined as evidence that the students made sense of the mathematics. ESM occurs if there is evidence that a concept is taken as shared, or more generally, if at any time during the episode a student extends or builds upon someone's thinking. *ESM focuses on what the students are doing.* Evidence of student mathematics occurs when the students provide evidence that they are making sense of the mathematics by bringing some "new" mathematics to the conversation. Consider the same excerpt as above except this time from the students' point of view.

Megan: So let's do the fraction first. How do we get rid of the fraction?
Jim: Times by the denominator.
Sue: Multiply by negative three.
Megan: Why did you pick negative three?
Sue: Because it's negative.
Megan: Because what's negative?

Sue: The fraction.
Megan: Because the fraction is negative. Okay. *(up to this point the student has only been clarifying their thinking and hence we see no building on student thinking)*
Sue: Oh, wait never mind. Don't times by a negative three. *(At this point we see the student change his mind. This seems to imply that by answering the ST's questions the student reflected on his statement/thought process, then found a flaw and corrected it. At this point this episode has become an ESM because the student is building on his own thinking)*
Megan: Why not?
Sue: Because then that way you can add it to the other side.
Megan: So you saw this right now when you said I don't want that number in front of x to be negative, and so we could get rid of it right now. But we're going to add this to the other side. So we want this to stay negative so when we add it will be positive. Does that make sense?

The amount of ESM in this research is limited. However, ESM was coded in a very similar way as BU, by counting the number of times students build on thinking. The previous example is therefore coded as an ESM-1.

ESM works as evidence that the ST is pushing students to develop a more robust understanding of mathematics. An ESM gives evidence that the students were pushed to change how they were thinking about the mathematics at hand. An ESM is also connected to the eliciting of student thinking because in order for an ESM to occur the students must share the change in their thinking. An ESM is part of effectively using student thinking because an effective use of student thinking requires a student to reason about mathematics. An ESM is evidence of students reasoning.

Well Used (WU)

WU cuts across both BU and ESM as a summative analysis of the ST and student interaction. The analysis for WU is done by answering the following questions: To what extent does the interaction between the ST and students give the students an opportunity to learn? Is someone making sense of the mathematics? Who is making sense of the mathematics? The students (better) or the ST (worse)? WU was measured on a five point scale: 0- not used; 1-

marginally used; 2- used; 3- well used; 4- extremely well used. In my data I have ample examples of numbers 0-2 and one or two examples of a number 3. I have no examples of a number 4 in this data. The following are more in depth descriptions/examples of each level of WU.

0- There are many ways not to use student mathematics. One seemingly new way I've seen in this research is a run with/do not use pairing. This discourse pattern occurs when the ST will pursue student thinking to the point that it is coded as a BU but soon after the ST abruptly moves on to something else. In this discourse pattern there is little chance for the students to learn because the ST moved on quickly without letting the students build on each others' thinking. Also included in this category is when the ST completely ignores students' comments. Within this discourse pattern there is not an opportunity for the students to learn which makes this discourse unproductive.

1- This level occurs when the ST marginally uses the students' mathematical thinking. Student thinking is minimized in the discussion and it is obvious that the ST is the one who is making sense of the mathematics and not the students. During this discourse pattern the ST might talk about what they made sense of to hopefully pass on the information to their students. Only one or two students may walk away from this lesson understanding what was going on. The students' mathematics was in some way used. The IRE cycle is included in this category along with Wood's (1998) funneling discourse pattern. These discourse patterns all have one thing in common. The ST could feel that they are using student thinking because they are responding to and "using" their students' mathematics. However, this is still an unproductive use of student thinking because the ST is doing the thinking instead of the students.

2- This level occurs when the ST is still driving the conversation, (dictating every turn in the conversation). However, the ST somewhat helps bring out student thinking, students take a part in the conversation and do part of the thinking. The students' thinking is present and used. (Generally a BU-1 and ESM-1 occurs)

3-This level occurs when the students are making sense of the mathematics. The students are in the majority in making sense of the mathematics. The ST's presence is still felt and the ST may feel the need to make sense of the mathematics for their students and tell them what they should have learned, however there is ample opportunity for the students to make sense of the mathematics and present counter arguments. (Generally a BU-2 and ESM-2 occurs)

4- This level occurs when the students are in control and making sense of the mathematics. The ST is not seen as the dispenser of knowledge. Instead the ST is there to help the students learn mathematics through pushing on their understanding. An example of this discourse pattern is focusing described by Wood (1998). In this discourse the ST is not a bystander, instead the ST pushes the students in a way that allows the students to make sense of the mathematics.

Thus WU gives an overall view of how well student thinking was used and if the classroom discourse that resulted created opportunities for students to deepen their understanding of mathematics. WU tells how effectively the student thinking was used.

Analysis of Three Episodes of Student Thinking

It is important to demonstrate how this discourse analysis was used and how it ties in to what occurred in the classroom. In this section I analyze three episodes of student thinking with my presented discourse analysis framework. Each episode was selected in order to show variety of uses of student thinking by the ST and to demonstrate the validity of the discourse analysis I used. The three selected episodes seem to provide sufficient information as to how the discourse analysis was used. Information about each episode of student thinking is given in six parts: background information, the transcribed episode, a play-by-play overview, the ST's perspective (BU), the students' perspectives (ESM), and how the student thinking was used (WU). This organization is done to help validate this discourse analysis in the eyes of the reader.

Excerpt from Megan 3-3-2011. Megan handed out a "warm up" at the beginning of class with a few review problems on it. This warm up was specifically about interpreting which lines better fit scatter plots and then writing the equation of the line of best fit. In this episode the ST already discussed which line best fit the data. In this case it was line B. The scatter plot and line in question are on an xy-plane which would allow for easy access to count and find the slope of line B. At this point in the lesson Megan is asking for the equation of line B.

Megan:	So what was the final equation that someone got for this? What was the equation of line B? I think I heard someone say it, but say it louder, the equation for the line.
Scott:	y=1x.
Megan:	Sorry I didn't hear you, did you say A=1x?
Scott:	y=1x.
Megan:	Oh, y=1x that makes sense. I was trying to figure out where the A came from. Okay Jonathan.
Jonathan:	Um, y=7/8x+0.
Megan:	Okay and these are both for line B, right?
Students:	Yeah.
Megan:	Okay, so how did you get, Scott how did you get y=1x?
Scott:	I don't know.
Megan:	You don't know?
Scott:	I guessed.
Megan:	You guessed, okay. I saw a lot of people had y=x or y=1x on their paper. It looks like when you're looking at this if you go from your y-intercept you go up one over one it looks like it crosses there pretty nicely, right? Um but I think, Jonathan how did you get the seven over eight x?
Jonathan:	I just found a point of part of the line where it crosses the section evenly, the most even, or closer and then I just, yeah.
Megan:	Okay, so what did you say Betsy?
Betsy:	Nothing.

Megan: Okay, so um if you look further down the line b, it starts getting further and further away from the intersection (the ST takes control and talks for a while)…

Play by play overview of this episode. Notice in the first few lines the ST misunderstands Scott who said y=1x. She then asks a clarifying question, and the student corrects her. This is not yet an example of BU because miscommunication in this sense is not seen as being significant in any mathematical way. After the miscommunication blurb Jonathan answers the ST's initial question with what seems as being the correct answer. At this point the ST decides to focus on Scott who said y=1x and ask him if how he got his answer. She tries to elicit some more thinking in order to better understand what he was thinking. Scott replies that he does not know, and after a little pushing he replies that he guessed. What is interesting here is that the ST decided to stop pursuing this train of thought. She could have asked "Well this seems to be a pretty reasonable guess. What did you do to make such a reasonable guess?" Really the possibilities here are endless, but here the ST decided to make sense of the situation and tell the students how she's thinking about the response y=1x. Which is, this is a common response and it seems fairly reasonable, but (which means it's incorrect) Jonathan, you tell us how you got 7x/8. Jonathan replies by saying, "I just found a point of part of the line where it crosses the section evenly, the most even, or closer and then I just, yeah." Then after a small distraction the ST assumes that Jonathan's thought process is exactly the same as hers so she takes it and makes sense of it for her students by explaining to the class why Jonathan's solution is better than the first solution given. The point when the ST takes over and starts making sense of the mathematics is an ideal opportunity for her to turn Jonathan's comment over to the class to help them make sense of what Jonathan was saying and why it might be more accurate. Unfortunately Megan takes advantage of this opportunity to tell the students what she thinking. There is no evidence that this is what the students think or that they are making sense of it. From this episode we only have evidence that the ST made sense of the mathematics.

From the ST's perspective. The ST did several good things in this episode. She asked clarifying questions and she got Jonathan to explain his solution strategy. From my coding above this episode is coded as a BU-1 for Jonathan's second comment. There was ample opportunity in this episode for the ST to better assess her students' current mathematical conceptions.

From the Students' perspective. Two solutions were shared by the students. One solution was briefly talked about by the ST and the other was explained by a student. In no way do the students' build on anyone's thinking. The students are simply responding to the ST. This episode is coded as an ESM-0 because ESM was not present in this episode.

Well Used. This episode is coded as a WU-1 *and* seems to be a show-and-tell episode. Some students present their solutions and the ST recognizes them as solutions, but the ST is *the one* making sense of the mathematics. In fact, later, just outside of the given transcription, the ST tells the students which equations are correct and which one is most accurate. The ST could have just told the students the correct answer in the first place and still had the same learning outcomes occur. At least in this case student thinking was somewhat used. The ST did have the students give their own solutions, which she then made sense of.

Excerpt of Megan 3-1-2011. Within this episode Megan just had the students get out their homework from last class. This assignment was designed to have the students practice writing linear equations in standard form. As part of this the ST wrote on the board a check list to make sure that an equation was in standard form. This list consisted of x, y on the same side; A must be positive; A, B and C must be whole numbers. In this particular episode Megan has been working out a problem with a given slope and y-intercept. She eventually writes the equation -3/4x+y=-2, then asks the following question:

Megan: Okay, is this in standard form?
Students: (shaking heads no)
Megan: No? Our x's and our y's are on the same side. What's wrong with it?
Sally: It's negative.
Megan: It's negative and what?
Sally: Because of the fraction.
Megan: And you have fractions right? Which one do you want to take care of first?
Students: Fraction.
Megan: Fraction, how do I get rid of the fractions?
Jonathan: Times by the denominator.
Alex: I know.
Megan: Yes.
Alex: Times it all by four.
Megan: Why four? I heard Jonathan say it earlier.
Alex: Because it's the denominator.
Megan: Because it's the denominator. So we're going to multiply both sides of the equation by four. It will take out this denominator. And multiply, why do we multiply by the denominator?
Jonathan: Because.
Megan: Because?
Sally: Because you do.
Megan: Because we do, yes? So if we have a multiply by four you can also look at it as a four over one. All of our integers all of our whole numbers can be written as a fraction like this. So if I do this multiplication I multiply straight across, right? We can also see that we have a four on the top and a four on the bottom, so we can cancel them. We can reduce in the first step before we multiply. Okay?

Play by play overview of this episode. Given the initial closed question "Is it in standard form?" the students respond by shaking their heads no. So the ST follows up with a more open question "What's wrong with it?" and Sally responds with "Its negative." Now notice how close-ended her follow up question is. "It's negative, and what?" By using Sally's thinking in this way the ST successfully changed her open question to a very closed question with the sole correct response being "because of the fraction" which Sally seems to be forced into giving such a response. Notice how the ST is in control of the thinking with the appearance of the students sharing information. In all actuality the students' comments are not making a big impact on the way this discussion is going. After Sally's fraction comment, the ST says "And you have fractions, right?" this is an interesting phrase, because in this phrase the ST tells the students that "fraction" was the answer she was looking for but it is done in such a peculiar way. "And we have fractions, right?" implies that it is a correct response. After that comment Megan asks which issue they should take care of first. The students respond that they want to get rid of the fractions first. So she responds with the question, "how do we get rid of the fractions?" The ST seemingly missed Jonathan's comment and responded to the Alex. With some probing, by the ST, Alex says that we should multiply it all by 4 because it's the denominator. The ST then asks the conceptually deep question "Why do we multiply by the denominator?" Then Sally responds with "Because you do," which shows little conceptual knowledge, and at this point the ST has the opportunity to elicit evidence from her students as to why they are doing what they "just do." This in turn would help the ST make sense of the students' mathematics, but instead she decides to explain her reasoning about why you can multiply to get rid of fractions. Then she moves on to a completely different idea and misses this opportunity.

From the ST's perspective. The ST received a lot of student comments throughout this episode, and the students seemed to respond to most of her questions. The ST did ask the clarifying question, "Why four?" which led her to better understand what one of her students was thinking. Hence this Episode was coded as a BU-1. Unfortunately most of the questions asked by the ST were very closed in nature. The nature of her questions led to limited student responses.

From the students' perspective. In this episode the students were expected to answer the ST's closed questions. The ST controlled the conversation to such an extent that the students did not need to build upon anyone's mathematics. Hence this episode is being coded as an ESM-0.

Well Used. Because of the dominant ST voice there was very little mathematics for the students to make sense of. However, when the student teacher asked the question "why four?" she wanted to see what this student was thinking. When the student replied with "because it's the denominator" the teacher then assumed that her student was thinking about the mathematics in the same way she was. This seems to be a questionable assumption, because there is no evidence that the student understands anything beyond the procedure of multiplying by the denominator. However, the ST made an interactive decision that somewhat used what the student said. This situation could have been made worse had the ST completely ignored the student's comment. Overall this episode brought about little of the students' mathematics, but the little that was brought up was somewhat used by the teacher. Hence, this episode is being coded as a WU-1.

Excerpt from Megan 3-24-2011. This lesson took place right at the end of the third term. So the STs and cooperating teacher decided that a filler day would allow students to catch up on late work while not giving them homework. This day's lesson was built around a scaling activity where the students were supposed to scale a picture down by one half. So the ST's created a lesson on scaling and scale factor. This excerpt is midway through the lesson were the ST is almost ready to present a grid method to scale down a picture. The ST displays a simple picture of a star in a square and then draws a square that is approximately one third the size of the original square. After completing the square she asks the following question:

Megan:	So I'm going to recreate this star in this box, what is the best way for me to do that? Does anybody know?
George:	With a ruler.
Megan:	With a ruler, okay what do you mean with a ruler? How would I use the ruler going to help me?
Heather:	You would use the centimeters.
Megan:	You use the centimeters. What do you mean, how would I use it?
Heather:	Um, just measure like each little um…
Megan:	Oh, so I could go in and measure and see how far down this is and then put that point there.

Heather: Yeah.
Megan: What other ways could do it without using a ruler?
Jonny: Meter stick.
Megan: Meter stick, other measuring tools. Just like we did remember in the map we had a grid over the top of it. We can create a grid on this picture as well. So if we create a grid, we can break down the picture into smaller pieces we can see a little bit easier where our picture should be. So I'm going to do the same thing. I'm going to create the same grid …

Play by play overview of this episode. In this episode the ST wants her students to learn how to scale pictures using a scale factor. The ST definitely thinks that the only way to use scale factor is through using grids. At this point in the discussion the students have not had any time to think about how they could actually scale a picture. So instead of asking the question "how could I scale down this star?" the ST asks "What is the best way for me to do that?" This tells the students that there is one particular solution that the ST is looking for, this is unfortunate because the students have not had time to think of a single way to scale a picture and now they're supposed to come up with the "best" way to actualize the scaling process. When George responds "with a ruler" I find it interesting that the ST decides to pursue this statement. There are at least two possible reasons why the ST would feel the need to pursue this student's statement. The first is that one could use a ruler in order to draw the grid lines (the strategy the ST is leaning towards using). Or second the ST may want to better understand what the student means by "with a ruler" did she mean to use a ruler as a measurement tool which measurements could then be scaled down. In all likelihood the ST could have thought of both possibilities as a reason to pursue this thought process. After this student's statement the ST decides to follow up on this thinking by asking "What do you mean with a ruler?" Heather responds by saying "You would use the centimeters…just like measure each little um." Now at this point the ST assumes that the student has a measurement strategy. So having "made sense" of the situation the Megan decides to explain what she thinks Heather means by saying, "Oh, so I could go in and measure and see how far down this is (a point on the original picture) and then put that point there (on the scaled picture) .” What I find interesting is that Megan leaves out the idea of using scale factor as a way to make the measurements from the original into the new scaled form. This idea is also completely absent from the interview she gave directly after this lesson. With this in mind I find it very likely that the Megan, in this case, never really considered measurement as a viable option that was worth exploring. Next Megan asks “What other way could we do it without using a ruler?” Then Jonny replies “meter stick.” So Megan switches into lecture mode and tells the students that they could use a grid. Then she uses a grid to scale the original picture down.

From the ST’s perspective. The ST does a pretty good job following up on George’s comment. Hence this episode is coded as a BU-2. However also note that the ST feels that with a couple of questions she can assume she understands what her students think. Then after having made an assumption she moves on as if her assumption is reality. There is still no evidence that when the ST explained what she thought her student was thinking the student was actually thinking about scaling and measurement in that way. It seems that in this case the ST made her assumptions a little too quickly. From the interview the ST decided that she moved on a little too quickly and she meant to point out that using measurement could work but it is too tedious to be worthwhile.

From the students’ perspective. In this episode the students were asked to explain their reasoning. However there was no building done by the students on any type of thinking. The ST did some building but the students never really participated. Hence this episode is coded as an ESM-0.

Well used. The ST pursued her students’ comments in a seemingly productive way until she abruptly changed topics and never came back to the idea of measurement. The students’ comments on using measurement to scale an object were not addressed at all. Hence, the students were not given an opportunity to change how they were thinking about measurement. This episode is being coded as a WU-0 (this is an example of a run with/do not use pairing).

Data Analysis of STs' Reasons

In this section I describe the grounded theory approach I used to analyze the STs' reasons with respect to using student thinking. I then describe the comparison I made between the observed discourse and the ST's reasons.

As part of my analysis, each post lesson interview was transcribed in its entirety. This included the STs' responses explaining why they used student thinking the way they did. Each ST response was condensed into the reasons given. Often multiple reasons were given for a single episode of student thinking. I printed out each reason on a slip of paper. With these reasons I used a grounded theory approach, which allowed me to group them according to the type of reason given. The reasons were then sorted a second time to ensure the accuracy of these groupings. Any discrepancies that occurred were discussed with a colleague until we both felt that the groupings were accurate. With this analysis completed, I analyzed the top types of reasons given and analyzed how these reasons were connected to different uses of student thinking. After this analysis, I compared the types of reasons given for every episode of student thinking with the discourse that resulted in that episode.

In order to compare the observed discourse and the STs' reasons, all of my data were compiled into two spreadsheets, one for Megan and the other for Olivia. In these spreadsheets each row represents one episode of student thinking. Each of the columns I through R represents a type of reason (e.g., "PLAN" represents a planned discussion, "MIS" represents a student's mathematical misconception—all of these types of reasons will be discussed in detail in the next chapter) and columns S, T and U represent the three parts of the discourse analysis. For the ST reasons each cell represents the total number of times that reason was used in the given episode of student thinking. Thus, if there is a 4 in the "ENG/DIS" column that means that four engagement or disposition reasons were used in that particular episode of student thinking (see Figure 2)

	A	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Megan	LENGTH	PLAN	JUX	ENG/DI	MIS	RET	TIME	MATH	LOSE	ASSESS	BU	ESM	WU
2	101	4	0	0	4	0	0	0	0	0	0	0	0	1
3	102	4	0	0	3	1	0	0	0	0	0	1	0	1
4	103	1	0	0	1	0	0	0	0	0	0	0	0	1
5	104	3	0	0	3	0	0	0	0	0	0	0	0	1
6	105	2	0	0	2	0	0	0	0	0	0	1	0	1
7	106	1	0	0	1	0	0	0	0	0	0	0	0	1
8	107	5	0	0	3	1	0	0	1	0	0	2	1	2
9	108	11	0	0	7	2	0	0	2	0	0	2	1	2
10	109	22	0	0	15	2	0	0	5	0	0	5	0	2

Figure 2. A picture of the spreadsheet created using Megan's reasons data.

Also note that column I which is titled "LENGTH" was added as part of this analysis. This column represents the number of times the ST responded to student thinking. Hence this column does not represent the length of an episode in minutes but instead it is a measure of the number of times the ST responded to student thinking on a specific topic.

Column A gives extra information for each episode. The number in the hundreds place represents the lesson (1 through 7) from which the episode was taken. The tens and ones place represent the episode in that lesson (01 up to 33). So the number 01 represents the first episode and 33 represents the thirty-third episode in that lesson. For example, the number 324 represents the twenty-fourth episode of student thinking taken from the third lesson.

After all the data were entered into the spreadsheets I used a conditional formatting tool to highlight the cells that were above average in each column. This created an easy way to see and visually interpret the data. For example, if column U was sorted from greatest to least then I could see that the values in column M tended to be above average when the values in column U were above average. This process of sorting and comparing above average values is what drove the analysis of comparing the STs' reasons with the observed classroom discourse.

CHAPTER 5: RESULTS

The results are presented in four sections that correspond to each of my research questions. First, to answer the question "What decisions do STs make with respect to using their students' mathematical thinking?" I give the results of an analysis of Megan's and Olivia's BU and ESM levels. I also compare and contrast Megan's and Olivia's classroom discourse by using BU, ESM and WU. By doing so I paint a better picture as to how Megan and Olivia individually used their students' thinking. Second, to answer the question, "How well do STs use their students' mathematical thinking?" I give the results of an analysis of Megan's and Olivia's WU levels. These results will show generalities in how well Megan and Olivia used their students thinking. Third, to answer the question "What reasons are behind STs' decisions with respect to using student thinking?" I give the results of an analysis of the reasons given by the STs with respect to the interactive decisions they made. This was a grounded theory analysis in which I used the reasons data to categorize and analyze the reasons. Fourth, to answer my final question "How do the STs' reasons for the use of student thinking influence students' opportunity to learn" I report the results of an analysis of the type of reasons given paired with the observed discourse. These results show how the reasons are connected to the observed classroom discourse and that some reasons, although well intentioned, often have undesirable results.

Megan's and Olivia's Use of their Students' Mathematical Thinking

These results describe how Megan and Olivia used their students thinking. I coded all of the episodes of student thinking from the lessons by using the BU, ESM and WU codes described in the methodology section and then compiled the data for each ST by lesson (see Table 1). This compilation shows the total of all of the BU, ESM and WU scores in each of the recorded lessons. For example, the 33 under Megan's WU on 2/25/2011 represents 0 WU-0's, 25

WU-1's and 4 WU-2's that occurred in that lesson. Thus there were 29 episodes in that particular lesson with a total WU score of 33 (0*0+25*1+4*2).

Table 1

ST Lessons' Discourse

	Megan				Olivia			
Date	BU	ESM	WU	# of Ep.	BU	ESM	WU	# of Ep.
2/25/2011	21	6	33	29	13	4	20	20
3/1/2011	17	3	34	33	9	6	23	23
3/3/2011	4	1	11	10	10	2	13	12
3/22/2011	17	7	17	16	5	0	12	13
3/24/2011	7	1	16	19	3	1	19	19
3/30/2011	14	5	32	30	2	5	30	30
4/1/2011	7	3	25	27	2	5	24	24
Total	87	26	168	164	44	23	141	141
Av. Per Ep.	0.53	0.16	1.02		0.31	0.16	1.00	

Note: BU represents Better Understanding, ESM represents Evidence of Student Mathematics, WU represents Well Used.

As a basic analysis of this data I summed the number of episodes and the codings of BU, ESM and WU in each lesson. I then calculated the total number of each as seen in Table 1. Given that BU, ESM and WU are different measurements that were coded in different ways, how meaningful the total of any coding is dependent upon the type of each individual coding. BU, for example, is essentially built off of counting the number of times a ST created the opportunity to better understand her students' thinking within an episode. Because BU counts the number of occurrences of Better Understanding it is logical to find the total number of occurrences within all seven lessons. ESM is similar to BU in that it counts the number of occurrences a student builds off of someone's thinking. Thus the total of ESM is also meaningful, because it lends insight as to how often over the seven observed lessons Evidence of Student Mathematics was present. On the other hand, the total of WU is not as useful because it is based off of a scale. To better understand why I am saying the sum of WU is inadequate, consider the following two fictitious data sets. In the first data set the ST had two episodes of student thinking and in both she received a WU-2. In the second set of data a different ST also had two episodes of student thinking. The first episode was coded with a WU-4 while the second was coded with a WU-0. So, the sum of both of these data sets totals to 4, but the teaching styles are very different. The first ST seems to be somewhat consistent with her interaction with her students, while the second ST seems to react very differently in different situations. Thus I argue that the total of WU yields too little information to be useful. Hence my analysis of WU will more heavily rely on a bar chart of the data and looking at the different number of occurrences of each level of WU. I should still point out that the average of WU per episode is somewhat useful because it can show a general trend over many episodes. The average WU per episode also adds insight to how generally the student thinking was used in each episode. Finally after finding the total of each coding, I then calculated the average number of codings.

How Megan and Olivia Created Opportunities to Make Sense of Student Thinking (BU)

This section helps answer my first research question: “What decisions do STs make with respect to using their students’ mathematical thinking?” This section is a description of how Megan and Olivia generally used student thinking to elicit more student thinking (BU).

In the majority of Megan’s and Olivia’s recorded episodes they did not elicit student thinking to a point that it would be considered a BU-1. In fact Megan obtained a BU-0 in 106 of the 164 recorded episodes of student thinking and Olivia obtained a BU-0 in 106 of the 141 recorded episodes of student thinking (see Figure 3).

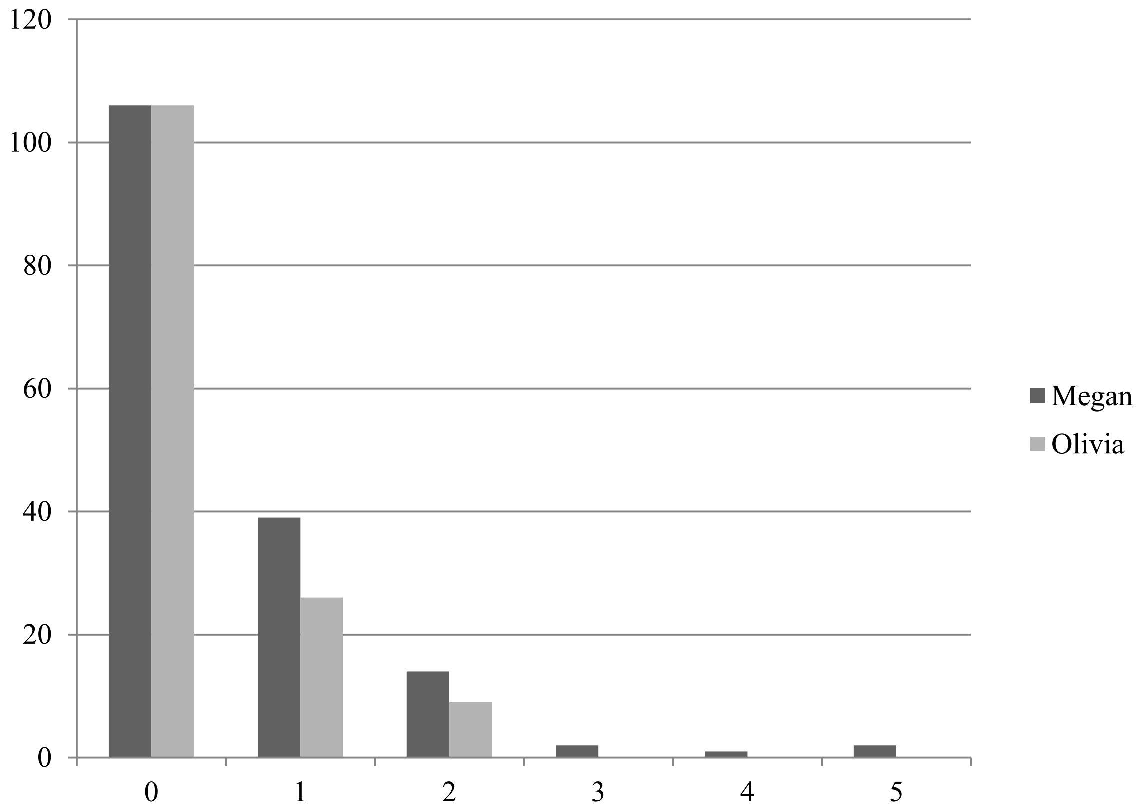


Figure 3. Bar chart displaying the number of recorded episodes and their level of BU.

Hence, there were many episodes where both STs did not use their students' thinking in a way that is consistent with an effective use. Hence, generally within an episode of student thinking both STs would ask questions of their students but they would not persue their students' thinking after the initial question. Most of the episodes of student thinking consisted of discourse patterns much like the I.R.E. cycle where the ST would ask a question, a student would respond, the ST would evaluate the response and then move on. This kind of cycle repeated itself regularly throughout both STs' lessons. It seems that both Megan and Olivia were generally satisfied with a classroom discourse that contained little pursuit of student thinking. Generally, Megan and Olivia did not elicit enough student thinking for either of them to really make sense of the student thinking, let alone build an adequate model of their students' current mathematical conceptions.

How Megan and Olivia Pushed On Student Thinking (ESM)

This section is tied to the last in that it helps answer my first question: "What decisions do STs make with respect to using their students' mathematical thinking?" This section is different in that instead of answering my first question by looking at BU, I am answering this question again but by looking at ESM. Hence, this section is a description of how Megan and Olivia generally pushed on student thinking and how often their students would respond with some evidence of student mathematics.

Megan and Olivia generally did not try to push on their students and have them respond with some evidence of student mathematics. In fact Megan obtained an ESM-0 in 143 of the 164 recorded episodes of student thinking and Olivia obtained an ESM-0 in 121 of the 141 recorded episodes of student thinking (see Figure 4).

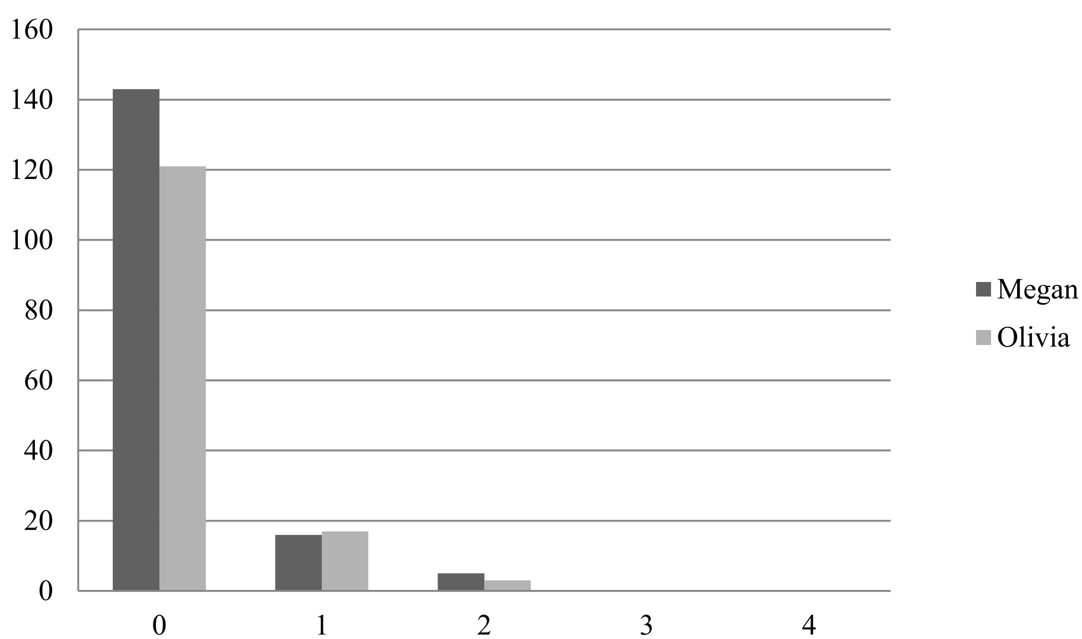


Figure 4. A bar graph displaying the number of recorded episodes and their level of EMS's

This means that the students generally did not show evidence of building on someone's mathematical thinking. Because we know that the students were generally not required to build off of someone's thinking then we can infer that Megan and Olivia, as the STs were the ones who were making sense of the mathematics and not the students. This lack of ESMs is problematic because it shows that the students were not being encouraged to think about and build on mathematics.

Comparing Megan and Olivia's Classroom Discourse

The last two sections seem to indicate that Megan and Olivia were at the same level of inadequate use of student thinking, but this was not the case. Megan and Olivia used student thinking differently. Hence, the following sections contain comparisons of Megan's classroom discourse and Olivia's classroom discourse. This comparison demonstrates how Megan and Olivia individually tended to use student thinking differently. In order to partially answer my first research question about how STs use student thinking, I analyzed how they as a student teaching pair used it in similar and in different ways. I made this comparison through BU and ESM; the results are given below.

Better Understanding. BU occurred close to twice as often in Megan's classroom than in Olivia's. In Megan's classroom a BU occurred about once out of every two episodes or 53% of the time. In Olivia's classroom a BU occurred about once out of every three episodes of student thinking or 31% of the time. This is an interesting finding because the lessons observed within this study covered the same material and, as mentioned previously, the STs planned each lesson together and taught their lessons in the same room. I feel it is therefore reasonable to conclude that the difference expressed here is due to the difference between each ST's interactive decisions and not differences in lesson plans or lesson materials. This appears to be a significant difference between the STs.

There are two simplified possibilities that could explain the difference in average BUs. The first possibility is that Megan could have made more opportunities to better understand her students in more episodes. This would imply that Megan would have almost twice as many lessons coded as a BU-1, BU-2 etc than Olivia did; Megan would also have about the same number of BU-0s as Olivia. The second possibility is that Megan could ask twice as many questions to better understand her students than compared to Olivia in similar episodes of student thinking, i.e. Megan received a BU-2 where Olivia received a BU-1 in an equivalent episode. If this were the case then for each BU-1 Olivia received Megan would receive the same number of corresponding BU-2. To help analyze the BU data I created a bar chart (see Figure 3).

Each data point on the chart is an episode of student thinking paired with the level of BU received in that episode. By looking at this chart you can see that both Megan and Olivia have the same number of BU-0s. Also note that Megan has higher occurrences in each BU level above 0. This higher occurrence is interesting, but it is not enough evidence to imply that the first possibility definitely occurred. On the other hand note that Megan's BUs taper off far more slowly than Olivia's, thus this gives plausibility to the second possibility.

In reality it is unrealistic to imply that only one or the other possibility occurred. In fact, individual episodes found in the data express both types of differences in BUs. The following is an example of Megan exploring deeper and pursuing student thinking more than Olivia in a similar episode.

Excerpts from Megan 2-25-2011 and Olivia 2-25-2011. These are the first lessons recorded as part of this study. In their post lesson interviews the STs mentioned that they did not prepare as much as they normally would for this lesson. This is easily seen because ideas were sequenced in different orders and different example problems were used between the two lessons. However, these lessons were still similar enough to allow comparison of episodes of student thinking within the two lessons. This lesson's objective was to help students learn how to write and re-write equations in standard form. As part of this lesson both STs wrote a list on the board which read: "Standard Form, Ax+By=C, X's and Y's on the same side, A is positive, No fractions." This list was referred to by the STs as a check list to see if an equation was in standard form.

Before delving into these episodes I must mention a slight confounding factor. Olivia taught every lesson before Megan did. The lessons that were recorded for these data included Olivia teaching 2nd hour and Megan teaching 3rd hour. Megan was present and helped Olivia's lessons in small ways like passing out papers and collecting homework. We could then imply that because Megan was participating in the lesson she could learn from Olivia's mistakes and then make changes to the lesson plan before she taught her lesson. This is probably true. However, Olivia also taught the same lesson 1st hour. Thus, Olivia had the opportunity, much like Megan, to see how the lesson went and then make revisions to her lesson before I observed it. Now I must ask the question who was more prepared? Was the ST who was able to teach the lesson once before I observed her or was it the ST who was able to watch and participate in the lesson twice before I observed her teach? At this time I am unable to conclude who was more prepared or how this difference in lesson preparation would affect the teaching outcomes. This was an uncontrollable factor that occurred because of my test subjects' schedule. Hence, I have noted this factor and assumed that this difference is negligible.

Megan 2-25-2011. After talking about standard form Megan gave an example problem that needed to be rewritten in standard form which she then worked out with the students. Then she decided to try another example. The following is the dialog that resulted.

Megan:	Let's do another one. Example two, we're gonna give you y=3x+4. How would I change this one into standard form?
Cade:	Minus 3x.
Megan:	Subtract 3x (writes it only on one side). Just like that?
Cade:	Yep. (1)
Megan:	Okay, so what do I get then?
Cade:	Oh no, you have to do it on the other side. (2)
Megan:	I do have to do it on the other side right? So I get -3x+y=4. Am I done?
Sara:	No.
Megan:	No, what's wrong?
Sara:	The three is negative not positive. (3)
Megan:	Awesome, we have that negative sign. How do we get rid of the negative sign?
Ben:	You add it.
Megan:	You add it? Okay, what would I add?
Sara:	No.
Megan:	No?
Sara:	Just change the signs.
Megan:	Just change the signs? What do you mean change the signs? So if we added it, then we'd get it back on the other side right? We just brought it over so we don't want to add it.
Sara:	Divide by one.
Sue:	Switch the signs.
Megan:	Divide by one or switch the signs? Why can't we just switch the signs?
Tim:	Because if you divide by one it does the same thing. (4)
Megan:	It's true. If we have negative three divided by a negative one then we have negative divided by a negative and that becomes a positive right? So we can definitely do that right?
Tim:	Yeah.
Megan:	I'm going to say is there any other operation we can do to get rid of that? We can divide by a negative one, is there anything else we can do to get rid of it?
Sue:	Times by one?
Megan:	We can times by 1 right? We can multiply. I'm going to use multiply because we're going to use that later. So I'm going to multiply this whole thing by negative one. Because if we do it to the -3x we have to do it to the y and we have to do it to this four right?
Sue:	So just change the signs.
Megan:	So you just change the signs. If you can see that you're just changing all the signs that works. Make sure you don't forget anything. So, we multiply this -1 in then what do we get?
Bill:	3x-y=-4.

Megan: Perfect. Okay, so now is this in standard form?
Cody: Yes.
Megan: Let's check it. The first point, x and y are on the same side? Check?
Cody: Yep.
Megan: A, B, and C are whole numbers? Do we have any fractions?
Bill: No.
Megan: No, and A is positive? We just fixed that right? So this is our standard form of the equation.

Throughout this episode Megan consistently asked for students to further explain themselves. As a result this episode was coded as a BU-4. To help point out this fact parenthetical numbers have been placed throughout this episode noting where students clarify their thinking. This type of discourse allowed Megan to interact with her students in a way that she could make sense of her students' mathematical thinking. Now we compare this episode to how Olivia reacts in a similar situation.

Olivia 2-25-2011. In this episode Olivia gave her students the equation y=5x+2 and asked them to write it in standard form. She then gave her students five minutes to work on it. This is the first of this problem type given in this lesson. This episode starts when the ST pulled the class back for a discussion.

Olivia: Okay, let's talk about it real quick. What was your first step? When you were doing this you wanted to get your x's and your y's on the same side what did you do?
Kathy: Minus two.
Olivia: Oh, you minused two. Why would you minus your two first?
Kathy: Yeah…
Olivia: You could do that. That would be fine. So let's go with it. Let's go with it. If you subtracted two, then you'd have to subtract two over here right?
Kathy: Yep.
Olivia: So then you would have y-2
Kathy: =5x.
Olivia: Okay, so what would you have to do from there?
Kathy: Um then you would uh…
Olivia: Okay, let's look at it k. From this equation right here (y-2=5x) let's look at our things that our special. Are our x's and y's on the same side?
Kathy: No.

Olivia: Nope, okay but do we have whole numbers? Are there any fractions?
Kathy: No fractions.
Olivia: Nope, and is our A positive? The number with the x is it positive?
Kathy: Yeah.
Olivia: Yep, so what's the only thing we need to fix and then it will be in standard form?
Kathy: Put the x and y on the same side.
Olivia: The x and the y on the same side. How can I get my x's and my y's on the same side?
Kathy: Minus 5x. (1)
Olivia: I could do that, I could minus my 5x then I'd have this (y-2) over on the same side too and have to switch it. I can do it in one simple step. What could I do? What can I move? Kaitlyn what did you say?
Brady: Minus y.
Olivia: You could minus your y, right? Do you see that? You could minus your y as well. So on this if I did that then I would end up with =2=5x-y. Then I just have it so I could rewrite that 5x-y=-2. But that way does that fit what I have?
Kathy: Yep.
Olivia: Are my x's and my y's on the same side?
Kathy: Yep.
Olivia: Do I have fractions?
Kathy: Nope.
Olivia: And is the number with the x positive?
Kathy: Yes.
Olivia: Yep, so we did it.

At the beginning of this episode Olivia attempted to have her student clarify their thinking when she asked "Oh, you minused 2, why would you minus your 2 first?" However, when her student failed to provide a quick response she ended up moving on before the student had a chance to respond. This lack of persistence allowed her student to get off the hook, and left Olivia unable to make sense of her student's thinking. After that point the ST failed, in all cases except for one, to ask questions and succeed in having her students respond in a way that she could have made sense of her students' mathematical thinking.

Megan Trying to Better Understand Her Students Where Olivia Chose Not to. The following episodes provide an example where Megan sought to better understand her students and Olivia chose not to. Both of these episodes were filmed on March 22, 2011. The STs had just given a test that covered material including finding slope when given two points, and writing different forms of linear equations. This class started with students practicing their multiplication skills for about five minutes. Then the STs handed out a paper with the five most missed problems from the last test. This was called an individual warm-up, and the students worked on it for about twenty minutes. These episodes are centered on the STs' work with the second problem, which reads, "2) a. Write the slope-intercept form of the line through (-1, 3) and (5, 15)."

Megan 3-22-2011. Megan and Olivia differed on a few aspects in this episode. In this case Megan decided to work out each problem acting as a scribe while her students told her what to do. The following is the dialog that resulted when they discussed problem 2:

Megan:	Number Two. It says write an equation in slope intercept form of the line through those two points. How do I do this?
Carlo:	y_2-y_1 over x_2-x_1.
Megan:	Awesome, so Carlo what is that y_2-y_1 (she finishes writing the equation)? And what is that you're telling me? What does that give us?
Carlo:	Slope. (1)
Megan:	It gives us the slope, right? So we have this $(y_2-y_1)/(x_2-x_1)$. To find our slope we just plug in our points up here this is our x_1, y_1, x_2, y_2. Fifteen minus our three all over five minus our negative one. So what's 15-3?
Sue:	12.
Megan:	Perfect. We had, well not so much in this class, but in the other classes that was a really big mistake, just subtracting. Okay, then 15-(-1)? [She meant to say 5-(-1)]
Steve:	6.
Megan:	6, right.

This episode was coded as a BU-1 because the student clarified his thinking when he answered the ST's question "what does that give us?" with "slope" (see the parenthetical 1). The question offered by the teacher is useful because it could help the ST better understand if the student did not know why he was doing that particular procedure or not.

Olivia 3-22-2011. Instead of being her students' scribe as Megan did, Olivia asked individual students to work out their solutions on the board. She then went over these solutions with the class. The following dialog resulted with the discussion of problem 2:

Olivia: So, I love how Sara set up her equation. Sara, how did you know to set up your equation like this?

Sally: It's the equation to get the slope from those two points is y_2-y_1 and x_2-x_1.

Olivia: So you can see that she plugged in, 15 was her y_2 and 3 was her y_1, and then x on the bottom. X_2, the 5 minus x_1 which was negative 1. (26:11) (no other student comments occurred in this episode)…

This episode is very short and was coded as a BU-0 because the ST did not ask for clarification from the student who worked out this particular problem.

These two episodes, even though they have the same subject matter, have two main differences that need to be addressed. The first is that each ST went about discussing this problem with very different approaches. Olivia chose to talk about how a student already solved a problem correctly. Before Olivia's student, Sara, explained her reasoning, she already knew that her solution was correct because Olivia said, "I love how Sara set up her equation." From this sentence Sara knew that not only was her answer correct but it was good enough to be praised. This probably led the class not to discuss this problem further because Sara got the answer right. What else would they talk about? Now contrast Olivia's approach to Megan's. Without writing anything on the board Megan chose to ask her students how to solve this problem. This difference leaves the question open, because the answer has not yet been given to the students. Megan's approach leaves more to be discussed because there is still uncertainty in how to work out the problem. This difference is probably why Megan's episode resulted in a BU-1 whereas Olivia's episode resulted in a BU-0.

A second difference is in the way the students first explained themselves. Olivia's student gave a more complete explanation. She did not only mention the equation she used, but she also said that she used it to find slope. Megan's student initially gave a partial solution; he only mentioned the equation and nothing else. This gave Megan more reason to ask the student to clarify his thinking.

In this case I believe that both differences contributed to Megan trying to better understand her student's current mathematical conceptions, whereas both differences contributed to Olivia not asking a clarification question. I used these episodes as an example because I want to point out that a different decision made by the ST and a different student comment greatly affected the BU outcome. Both STs handled the situations "appropriately" but it was they who created the situations in the first place and these situations were more and less beneficial in better understanding the students' thinking.

Evidence of Student Mathematics. To better understand the spread of the ESM data I created a bar chart with each data point coming from an episode of student thinking being paired with the ESM score received in that episode. This chart is entitled "Spread of ESM's"(see Figure 4). One interesting note is that the shape of the data in the bar chart is very similar. They both have a large number of ESM-0 which then tapers off quite dramatically. This shows that ESM basically occurred at the same rate in both Megan's and Olivia's classes.

Evidence of Student Mathematics (ESM) had very different results than BU. I found it to be very interesting that ESM occurred basically the same number of times in both classrooms (both occurred in 16% of the recorded episodes). This seems to point out that ESM and BU occur independent of each other. I find this to be most interesting because when I built these models of BU and ESM I thought that they would be somewhat related. It would seem logical that the better the ST seeks to make sense of their students' mathematics the more often the ST would have the students build off of each other's thinking. However, it seems that these STs, whether succeeding in better understand their students or not, were not pushing students to build off of each other's thinking. So, at least at this point it seems that there is no correlation between BU and ESM. One thing to take away from this is that the STs were cultivating (or not cultivating)

ESM in a very similar manner. I hypothesize that this occurred because the cooperating teacher setup a classroom culture that was not conducive to students building upon other's thinking. The cooperating teacher taught and had his classroom set up in a very traditional manner. Then this classroom culture bled into the STs' classroom which hindered students building upon other's thinking. A second reason could be that both STs believed (as expressed during a final interview with both STs) that their students, who were in a remedial class, could not reason deeply about mathematics. This belief would lead the STs not to promote students to build off of other's thinking.

To illustrate this idea consider the penultimate excerpts from the BU comparison. In both lengthy episodes there is only one occurrence of ESM. It occurred in the beginning of Megan's episode when Cameron corrected both his own words and the ST's intentional misunderstanding. See the parenthetical asterisk in the following excerpt (from Megan 2-25-2011).

> Megan: Let's do another one. Example 2, we're going to give you y=3x+4. How would I change this one into standard form?
> Cameron: Minus 3x.
> Megan: Subtract 3x (writes it only on one side). Just like that?
> Cameron: Yep.
> Megan: Okay, so what do I get then?
> Cameron: Oh no, you have to do it on the other side. (*)
> Megan: I do have to do it on the other side, right? So I get -3x+y=4. Am I done?...

This entire episode was coded as an ESM-1 and BU-4 whereas Olivia's episode was coded at an ESM-0 and a BU-1. The difference in the BU is significant while the difference in the ESM is not. The fact that only one ESM occurred in both of these episodes, which had very different BU codings, again points to the idea that the STs' ability to create opportunities to better understand their students and their ability to promote students to build off of other's thinking seem to be unrelated.

How Well Megan and Olivia Used Student Thinking (WU)

This section helps answer my second question: “How well do STs use their students’ mathematical thinking?” This section is broken up into three segments regarding Well Used. The first segment is a description of how well Megan generally used student thinking. The second segment is a description of how well Olivia generally used student thinking. The third segment is a comparison of Megan and Olivia’s WU scores. This comparison lends insight as to how well Megan and Olivia used student thinking.

Megan received a WU-1 in 145 of the total 164 recorded episodes of student thinking. This means that in the vast majority of episodes of student thinking Megan marginally used her students’ thinking. Megan marginally used her students’ thinking by responding to her students’ comments and having a class discussion while ineffectively using the student thinking. In Megan’s case she would try to use student thinking. However, her use of student thinking did not give students an opportunity to learn. Instead, Megan generally was the one who was making sense of the mathematics while her students seemed to be passive listeners. For example, Megan would often repeat what students would say (clearly a use of student thinking), however, the use of student thinking stopped there.

Olivia received a WU-1 in 135 of the total 141 recorded episodes of student thinking. This means that in most of the episodes of student thinking Olivia also somewhat used her students’ thinking. Olivia somewhat used her students’ thinking by generally responding to her students’ comment. In Olivia’s case she would generally use the student thinking by explaining a students’ thinking to the class. Thus Olivia generally would take the opportunity to learn from her students as she made sense of the mathematics and attempted to deliver her understanding of the mathematics to her class via an explanation.

To better understand how well Megan and Olivia individually used student thinking I compared their WU scores. The average of Well Used (WU) turned out to be pretty close between the STs, with Megan averaging a 1.02 on the WU scale and Olivia averaging 1.00. This difference was puzzling because of the fact that Megan had a higher BU and an equivalent ESM so why would she be so close on the WU scale. So to help me analyze this similarity I graphed the number of occurrences of each level of WU on the following bar graph.

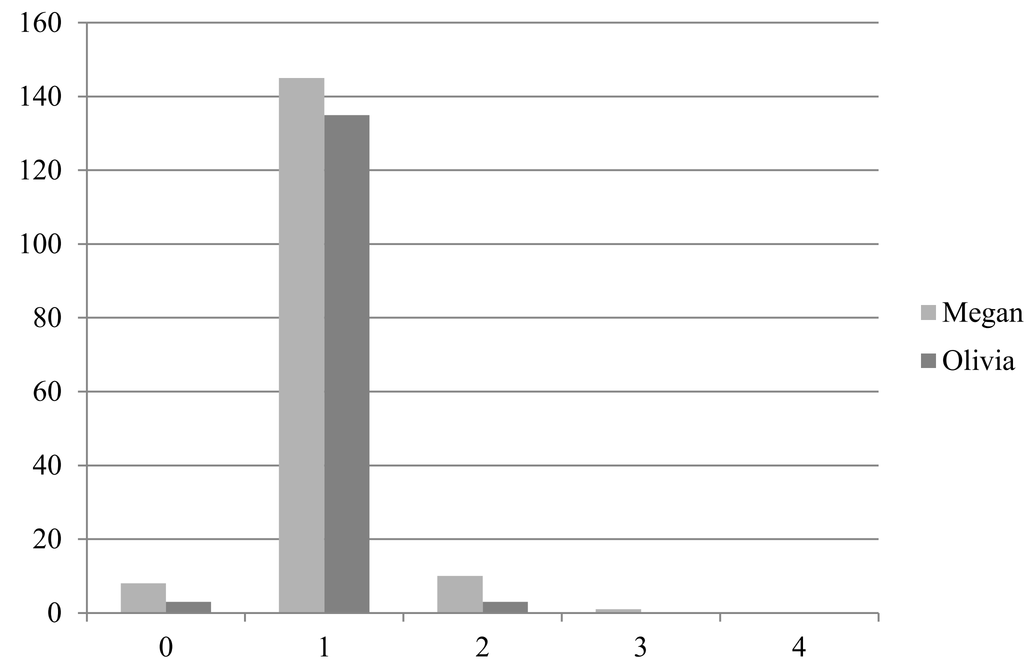


Figure 5. A bar graph displaying the number of recorded episodes and their level of WU's.

As shown in Figure 5 Megan has more occurrences in all levels of WU. This shows that Megan's distribution is centered about 1 but it is more varied while Olivia's distribution is tightly centered about 1. This shows that Megan and Olivia's distributions are similar in that they center about WU-1, but differ in that Olivia's is more tightly distributed about WU-1. On average both STs tended to use student thinking to the same degree, WU-1. However, Megan's larger spread of data indicates that she had a more varied approach to how she used student thinking, while Olivia's tighter spread of data indicates that she was very consistent in how she used student thinking. Olivia's consistency then implies that both STs used student thinking in similar (centered around WU-1) but yet different ways (more varied approach vs. less varied approach). The following are examples of how Olivia and Megan used student thinking differently.

The following is an excerpt from Megan's lesson on March 22, 2011. This is an excerpt from the same lesson where the STs had students work out the most missed problems on the test they took earlier in the week. The question that was given on the test gave the equations of four lines (labeled a, b, c, and d) in standard form and asked students to find which line was not parallel to the other three? However, the question the STs gave to their students to work out in class just asked for the students to rewrite the equations in slope-intercept form. After the students had given all of the equations of the four lines in slope-intercept form Megan asked the original test question. The following dialog resulted:

Megan 3-22-2011

Megan:	If I were to ask you a question on this. We want to look for which of these lines are parallel? And which line is not parallel? Think about that for a second. If I tell you, of these four equations can you tell which ones are parallel and which ones aren't parallel?
Joe:	That's easy.
Megan:	Why is it easy?
Joe:	Oh, it's easy. Its B and D. (1)
Megan:	B and D are what?
Joe:	Parallel. (2)
Megan:	B and D are parallel? Okay, why?
Steve:	No, it's B, C, and D. (3*)
Megan:	Oh, so it's B, C and D? Do you agree Jonathan, or no?
Jonathan:	Yeah, sure. (4)
Steph:	And A.
Ss:	No, because A is negative. (*)
Megan:	Why not A? Say it a little louder.
Jordan:	A's negative.
Megan:	What's negative in A? Okay guys stay with me. Shhh. What's negative in A?
Jordan:	The three. (*)

Megan: The three, right? So when we say we have parallel lines what do we know is going to be in common?
Jesse: The slope.
Megan: The slope, right?

This episode was coded as a BU-5, ESM-2 and WU-3.

The following is the same episode from Olivia.

Olivia: Okay, so question, what is, um… Well, I'll just ask this one first. So, looking at those equations we had: y=-3x+3, y=3x+9, y=3x+9/4 and y=3x-3. Okay, three of those lines are parallel to each other and one line is not. Which one is the one, which line is not parallel to the other lines? Kalyn? (23:32)
Janet: A.
Olivia: A, what made you say A.
Janet: Because it's negative three. (1)
Olivia: Because it's negative three? So, how do you know? Just because the slope was negative three that it wasn't parallel?
Janet: All of the other ones were three. (2)
Olivia: Because all of the other ones were three and what do we know about parallel lines? I know we've been finding slopes of lines parallel…

This episode was coded as a BU-2, ESM-0 and WU-1.

It seems that the key difference between these two episodes is the STs' initial question. Megan started off with a more open-ended question than Olivia did. Megan asked "Of these four equations can you tell which ones are parallel and which ones aren't parallel?" Olivia asked "Three of those lines are parallel to each other and one line is not. Which one is the one—which line is not parallel to the other lines?" Both questions were answered according to how open they were. Megan's class had a conversation about which of the three lines were parallel, where Olivia's class did not. However, the last segment of both episodes is almost exactly the same. One student from both Olivia's and Megan's classes explained fairly quickly why line *A* was not parallel to the other three lines.

The direct comparison of these two episodes is probably not fair to both STs because the general trend with respect to WU was that overall the STs used or failed to use student thinking in very similar ways. These episodes were used as an example to illustrate the difference in spread of Megan as an individual in using student thinking compared to Olivia. The discourse style seen in Megan's episode did not occur on a regular basis. It was just needed to show that this level of WU was at least present even if it was a rarity.

STs' Reasons

This section helps answer my third research question: What are the reasons behind STs' decisions with respect to using student thinking? During the post lesson interviews I asked the STs why they chose to react to student thinking in the ways they did. The number of episodes I discussed with each participant was about the same. I discussed three more episodes with Megan than with Olivia totaling to 35 episodes with Meagan and 32 episodes with Olivia. I see this difference as being negligible because I collected similar amounts of data on which to base my findings. These data represent 67 different episodes of student thinking where a ST disclosed why they chose to do what they did. These 67 episodes are not a random sample of all of the episodes I observed. In fact, I focused on selecting episodes of student thinking where the ST's decision seemed to be affected by mathematics at hand, or where I saw seemingly unexplainable teaching decisions. After having asked about specific reasons why the STs made a decision I avoided asking about repeat scenarios. I did this because I concluded that I already knew why they made that decision, so I did not need to discuss it with them again. Hence, these 67 episodes of student thinking are not an accurate representation of how the STs used student thinking throughout the observed lessons. Table 2 shows each type of reason and its frequency.

Table 2

Categorized ST Reasons

Reasons	Number of Occurrences
Mathematics−Background	38
Mathematics−Student Misconception	11
Student Engagement or Disposition	17
Helping with Retention	6
Liked Comparing Two Ideas	6
I Did Not Want to Lose Anyone	6
Formative Assessment	4
Planned Discussion	3
Class Time Remaining	3
Total	94

Table 2 is helpful because it lists the reasons that were given most often to least often. A description of each category follows with one or two examples of each reason category.

Mathematical reasons made up the greatest part of the reasons given by both STs, which was roughly 52% of the reasons given. Mathematics has two subcategories: Background Mathematics and Student Misconception. The background mathematics subcategory made up roughly 40% of all of the reasons given. This category contains reasons that are mathematical in nature but do not contain elements about students or pedagogy. For example, when I asked, “A student had the question of what does the ‘or equal to’ mean in an inequality statement you decided to talk about it. Why?” two reasons were given. The first was that “this topic was important to talk about because it has application in future mathematics.” The second reason was “I find this topic very interesting mathematically.” Both of these reasons do not describe how a student or students are missing a key concept, but yet they are still mathematical in nature. Another example is when I asked, “Eric used a different first step to write an equation in standard form, and you decided to pursue his thinking. Why?" The ST responded, "It still requires the same amount of steps, but I liked the idea that you could avoid the negative A." Thus we can see that mathematics was, for these STs, the most dominant aspect that affected these teaching decisions.

The second subcategory of Mathematics was Student Misconception. These types of reasons occurred eleven times or about 12% of the recorded reasons. This category is best introduced through the following examples: "There was a misconception going on where the students would multiply by the two denominators added together and I wanted to clarify that misconception;" and "When we worked with slope intercept form we often multiplied by the reciprocal, and students think that is the normal way to get rid of fractions [in an equation]." As seen in these previous examples the STs understood that the students had a misconception so the STs reacted in a way to hopefully help their students correct the misconception. The key difference between this category and a Background Mathematics reason is that in this category students were doing something wrong, where in Background Mathematics reasons the students were doing something right.

The second largest category was Student Engagement or Disposition. This category contains reasons in which the ST tried to get students to engage and participate more in the lesson. There were 17 recorded occurrences of this reason, or about 18% of the reasons given. It is obvious that these STs were concerned with student engagement and participation. This could be related to the fact that most of the time both STs and the cooperating teacher were present in the classroom. This created an environment where the students sat still and were quiet. However there were only a few students who actively participated in the lesson. Thus, because there were not a lot of students sharing their ideas, the STs saw the need to get more students engaged in the lesson. Hence, student engagement affected the STs' decisions. One engagement reason given by both STs was that they often repeated student comments because they wanted to make sure the other students heard the comment so more students could engage in the lesson. The following is a second example of an "engaging more students" reason. I asked, "Two students were having a heated debate, then you decided to have a class vote. Why? "I had a class vote to get more students involved in the discussion."

The third category was Helping with Retention. This reason occurred six times or about 6% of the recorded reasons. One thing that was interesting about this category is that the STs had good intentions in trying to help the students with retention. However, the most often occurring decision made by the STs to help with retention was to repeat themselves or a student. These actions tended to be unproductive as far as retention was concerned (this topic will be discussed further in the next section). The following are two examples of a retention reason:

J: Several times you had your students read aloud inequality statements. Why?
ST: Just more reiteration.

J: You had trouble understanding what your student meant by what they were saying, but when you figured out what the student was saying, you decided to talk about it. Why?
ST: These students really struggle with retention. I hope that by repeating an idea it will help with retention.

The fourth category was Liked Comparing Two Ideas. This reason occurred six times or about 6% of the recorded reasons. These reasons are related to the idea that it is good to solve problems in multiple ways. However, these reasons are not related to the mathematics that results from solving a problem in two different ways. Instead, this stems from a ST belief that it's good to point out multiple ways to solve a problem (as opposed to pointing out the mathematics that comes out of those multiple strategies). The following are examples of the ST liked comparing two ideas: “I liked having two different ways to solve the problem” and “I wanted to make sure that the students knew that it was okay to work out the problem using either method.”

The fifth category was I Did Not Want to Lose Anyone. This reason occurred six times or about 6% of the recorded reasons. This is another example of the STs doing something that I found to be somewhat unproductive with good intentions. In this case the ST would either pursue student thinking that did not have potential or they would not pursue student thinking because it was too difficult for the rest of the class to follow. The following are two examples of the “I did not want to lose anyone” reason:

> J: Frank said, “multiply the denominator by 5” which was incorrect. You perused his thinking to help him clarify what he actually meant. Why?
>
> ST: I went with this comment not because this student would actually multiply the denominator by five, but because if another student were lost and heard the incorrect statement he might become confused.

> J: A student gave an answer to a fraction multiplication problem and you said “well how did you find the answer so fast?” Why did you decide to bring that up?”
>
> ST: I didn’t want to lose anybody.

The sixth category was Formative Assessment. This reason occurred four times or roughly 4% of the recorded reasons. This category contained reasons that were often used when the ST would ask for information from the students but then not use it. I found it interesting how the STs did these formative assessments but the information seen did not really affect the lesson (this topic will also be covered in the next section). The following are two examples of formative assessment reasons:

> J: You brought up a bunch of student thinking asking about absolute value, then you didn’t use any thing the students said. Why?
>
> ST: I guess that I was just getting feedback then, not analyzing what was being said. I would have liked to talk about it. I just wanted to see what the students knew about absolute value, because they haven’t talked about it since we came here in January.

The seventh category was Planned Discussion. This reason occurred three times or roughly 3% of the observed reasons. This category ties for the smallest categorized group of reasons. This category consists of the STs already planning on talking about a particular topic. The following is a planned discussion reason:

J: At the beginning of class you had a discussion about parallel lines in relation to a recent test problem. Why did you do that?

ST: I planned on talking about this. Most of our students missed it on the quiz, so there was a need.

The eighth category was Class Time Remaining. This reason occurred three times or roughly 3% of the observed reasons. This category ties for the smallest categorized group of reasons. How small of a factor time played in these STs' decisions seems to be related to the fact that the class that was observed in this study had no set schedule and therefore there was no need to cover some amount of material in order to prepare the students for the end of level test. The following are two examples of these types of reasons:

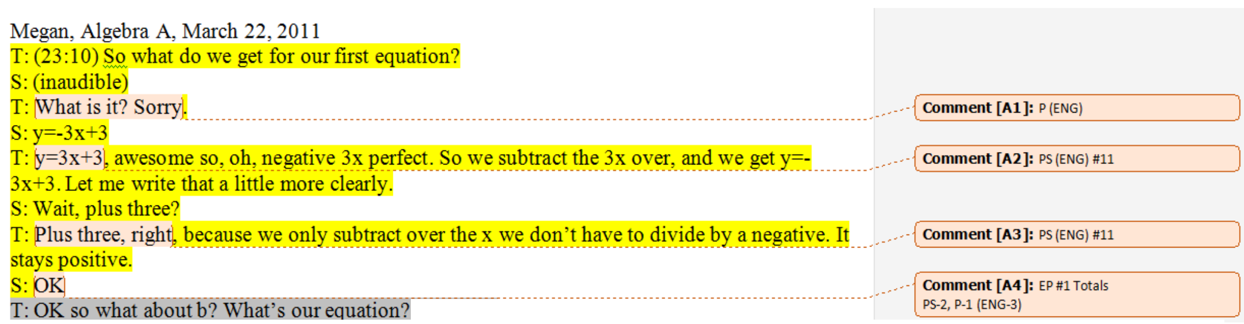
J: Today you relied a lot on Jonathan, where before you chose not to. Why did you do this?

ST: I was short on time so I was willing to ask students who would give me a fast answer.

J: At the end of class David came to the board and wrote an incorrect answer, you ended up stepping in and correcting it. Why?

ST: … and time played a little bit of a factor.

These categorized reasons are helpful in that they show what reasons the STs were basing their actions upon. However, the quantity and percents I have listed with each reason is somewhat meaningless because it only shows how often I, as the researcher, asked questions about different episodes of student thinking. This quantity does not give a general perspective on how the STs' reasons were used to make interactive decisions. In order to produce a more general perspective I revisited the transcribed lessons. Every time the ST responded to a student comment it was coded with the most likely reason why the ST did so. This coding was done by finding a similar situation in which the ST gave a reason to why they responded to student thinking the way they did. For example, I asked Megan "Why do you restate what students say?" She responded "I restate what students say to make sure everyone hears and to get students to focus on a new idea." So every time Megan repeated her students I matched that response with the given engagement reason. To make it easier to match reasons with responses I numbered all of the reasons. The reason given above was reason number 11. Figure 6 is an example of this coding.


Megan, Algebra A, March 22, 2011
T: (23:10) So what do we get for our first equation?
S: (inaudible)
T: What is it? Sorry.
S: y=-3x+3
T: y=3x+3, awesome so, oh, negative 3x perfect. So we subtract the 3x over, and we get y=-3x+3. Let me write that a little more clearly.
S: Wait, plus three?
T: Plus three, right, because we only subtract over the x we don't have to divide by a negative. It stays positive.
S: OK
T: OK so what about b? What's our equation?

Comment [A1]: P (ENG)
Comment [A2]: PS (ENG) #11
Comment [A3]: PS (ENG) #11
Comment [A4]: EP #1 Totals PS-2, P-1 (ENG-3)

Figure 6. An example of how the reasons for using student thinking were assigned to transcribed episodes.

In the second and third responses Megan decided to repeat the student comments. The "(ENG)" means that it is an engagement reason and "#11" means that it is Megan's eleventh reason. Now note that the first response does not have a numbered reason assigned to it. This is because I never asked Megan, "Why do you have students repeat what they said when you did not hear or understand them the first time they said it?" This means that I had to make my best guess as to why Megan responded the way she did. After considering how concerned Megan is with student engagement, and how non-engaging it would be not to address a student comment because she did not know what the comment was, I decided that in the very least Megan was concerned with engaging her students with her first response.

After each code was given these reasons were totaled for each episode. The figure above is an example of the first episode of student thinking on March 22, 2011. Table 3 consists of all of the reasons data that were collected in this way.

Table 3

Reasons Data	Total Number of Occurrences	
	Megan	Olivia
Student Engagement or Disposition	628	394
Mathematics	129	38
Student Misconception	66	56
I Did Not Want to Lose Anyone	28	14
Helping with Retention	20	122
Planned Discussion	10	4
Liked Comparing Two Ideas	6	2
Class Time Remaining	6	0
Formative Assessment	0	29

The first thing to note is that both STs used Student Engagement or Disposition to affect most of their interactive decisions, with Megan totaling 628 and Olivia 394. No other category for either ST comes close to these numbers. This shows that these STs were not primarily concerned with students' mathematical thinking. Instead, they were more concerned with having the students participating and listening to the lesson. I must point out that most of these reasons were paired with the STs' interactive decision of repeating what a student just said. There was never any consistent evidence that the STs were actually engaging the students by repeating a student statement. However, the STs obviously thought they were because they continually did it. This is a prime example of having good intentions that yield poor results. Evidence of STs having good intentions that yield poor results is discussed in the next section.

The next biggest categories vary between the STs. Megan's second largest category is Mathematics and Olivia's is Helping with Retention. Both of these reasons occurred a little less than once per episode of student thinking. This difference illustrates an important difference between the STs approaches to teaching mathematics. Megan was more often concerned with the mathematics, which led her to pursue student thinking which then helped her Better Understand the student thinking. Olivia was more concerned with retention, which led her to not only repeat a student statement but to also talk about what the student meant and what it implied in her own words. She did this so the students could hear the reasoning again and this time retain the information. This is another case where good intentions yielded poor results.

The third biggest category was Student Misconception. A misconception reason occurred at the same rate for both STs at two misconception reasons in five episodes of student thinking. It is interesting that it happened at the same rate. Also, it seems that in episodes of student thinking with student misconception reasons the STs generally used their students' thinking in more productive ways than episodes with other reasons.

The only other category that is large enough to merit attention is Olivia's Formative Assessment category. This occurred at a rate of one in every three episodes of student thinking. This would occur when Olivia would bring up student thinking in something like a brain storming session and then not use it. She did this because she wanted to see or assess what the students knew and not pursue their thinking or thought processes. It is problematic to cultivate student comments and then seldom use them. This practice could convey to students that what they have to say is not needed and is unimportant. I see this as a discourse that hurts student engagement and disposition. Still, it is important to assess students' knowledge, but assessing student knowledge in this way yields poor results. This is my final example of good intentions leading to poor results.

Results of Comparing Observed Discourse and ST's Reasons

This section helps answer my fourth research question: How do the STs' reasons for the use of student thinking influence students' opportunity to learn? This section is organized into two parts. First is the comparison of Megan's observed discourse and her reasons. Second the same comparison will be done for Olivia's observed discourse and her reasons. This analysis was done to see how different reasons for using student thinking were connected uses of student thinking that created opportunities for students to learn.

Comparison of Megan's Observed Discourse and her Reasons

This analysis revealed an interesting pattern in how Megan used student thinking. By sorting the "well used" column (Column U) from greatest to least I found that for the 11 episodes that had either WU-3 or WU-2 the following reasons or attributes also tended to be above average: Length, Engagement or Disposition, Student Misconception and Mathematics (see Figure 7).

	A	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Megan	LENGTH	PLAN	JUX	ENG/DI	MIS	RET	TIME	MATH	LOSE	ASSESS	BU	ESM	WU
2	405	10	1	0	4	5	5	0	5	0	0	5	2	3
3	107	5	0	0	3	1	0	0	1	0	0	2	1	2
4	108	11	0	0	7	2	0	0	2	0	0	2	1	2
5	109	22	0	0	15	2	0	0	5	0	0	5	0	2
6	112	14	0	0	7	4	0	0	3	0	0	3	1	2
7	211	8	0	0	4	2	0	0	2	0	0	3	1	2
8	224	7	0	0	5	1	0	0	1	0	0	2	2	2
9	303	8	0	0	6	1	0	0	1	0	0	1	1	2
10	602	13	2	0	8	3	0	0	0	0	0	2	1	2
11	624	7	0	0	4	2	0	0	1	0	0	0	1	2
12	723	10	0	0	6	0	0	0	3	1	0	2	2	2
13	101	4	0	0	4	0	0	0	0	0	0	0	0	1

Figure 7. A picture of the spread sheet that was used to analyze Megan's observed discourse patterns.

In order to find the strength of these four columns in predicting how well the STs used their students' mathematical thinking I compared the number of episodes in each reason column that contained above average values to the number of WU-2s and WU-3s that are found in those episodes. This creates a ratio which helps show the strength of the relationship between a reason and WU.

Student Engagement or Disposition (Column L) seems to predict how well Megan used her students' thinking. This relationship is problematic because these reasons were used by both STs to justify repeating student comments (which I have characterized, in general, as an unproductive use of student thinking). So, then why would above average unproductive uses of student thinking lead to better use of student thinking? When discussing this problem with a colleague I realized a connection. Engagement and disposition reasons were overwhelmingly present in most episodes. Hence, engagement and dispositions reasons cannot be analyzed by focusing on whether they are present or absent, because they are almost always present. However, this category gains meaning in that it is a somewhat accurate representation of the length of the episode, meaning the number of times Megan responded to a student comment. So, I calculated the length of each episode to see if it shared the same information with the engagement and dispositions reasons. Thus, I added the "LENGTH" column to the spreadsheets. I did this because engagement and disposition reasons produce unproductive uses of student thinking and when these reasons are totaled by episode they seem to represent the length of the episode. By comparing the "LENGTH" column and the "ENG/DIS" column I found that both of these columns were above average for all of the episodes coded WU-2 and WU-3. Also, for the most part, when the episode length is above average, engagement and disposition reasons are also above average. Hence, both of these columns contain basically the same information. Thus, I did not analyze engagement and disposition reasons and instead I analyzed the length of each episode.

In the Length column there were 64 episodes out of 164 that were above the average length of 5.3 ST responses per episode. Of these 64 episodes 10 of them were either WU-2 or WU-3. Hence I can conclude that when Megan would choose to stay focused on a topic longer than normal, one out of six times, or 15.6% of the time, she would better use her students' thinking. This shows that when Megan took the initiative to stay on a particular topic sometimes she would end up using student thinking in more effective ways. I find this interesting because choosing how long to stay on a topic is an easy action to carry out as a ST. Also, it is important to realize that staying on one topic for too long could potentially be very boring for the students if there are not sufficiently important ideas to discuss. However, this analysis does show that staying on topic a little bit longer is potentially beneficial to STs.

In the Student Misconception column there were 37 episodes out of the 164 that were above the average of 0.4 occurrences per episode. This average is particularly nice because instead of focusing on the number of episodes that were above average we can focus on when student misconceptions were present or not, because in this case it is the same thing. So, out of the 164 episodes 37 of them had at least one student misconception reason. Of these 37 episodes 10 of them were either WU-2 or WU-3. Hence I can conclude that, when Megan's reason for using student thinking was to respond to a student misconception, 27% of the time she would use the student thinking in more effective ways than normal. This positive association could be attributed to the fact that when a student has an obvious misconception it is rare for the ST to assume otherwise. A student misconception also tends to imply that this student does not get it and that what has been done before (telling) did not help them learn it. So, the ST tends to see the need to push on their student's thinking in order to help the student correct their misconception. For Megan, a student misconception seems to be the most recognizable form of student thinking that needs to be addressed.

In the Mathematics column there were 69 episodes out of the 164 that were above the average of 0.79 occurrences per episode. This average is also nice because by comparing above average to below average we are also comparing the presence or absence of a mathematical reason. Of the 69 episodes with a mathematical reason present 10 of them were either WU-2 or WU-3. Hence I can conclude that when Megan used a mathematical reason to justify her use of student thinking 14.5% of the time she used the student thinking in a more effective way.

Thus Student Misconception was the most productive reason for Megan to use student thinking, then Episode Length and Mathematics. I see all three of these reasons as being good intentions that actually lead to somewhat positive results. Responding to a student misconception, pursuing a concept for a considerable length of time, and using mathematics as a tool to base decisions of off are all very good reasons to be considering when teaching mathematics.

Comparison of Olivia's Observed Discourse and her Reasons

Using the same analysis as above, I analyzed Olivia's data by sorting column U from greatest to least. By doing so I found the following results (see Figure 8).

	A	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Olivia	LENGTH	PLAN	JUX	ENG/DI	MIS	RET	TIME	MATH	LOSE	ASSESS	BU	ESM	WU
2	112	15	0	1	10	3	1	0	0	0	2	2	1	2
3	203	11	0	0	8	2	0	0	1	0	0	2	1	2
4	304	7	0	0	6	0	0	0	1	0	1	2	1	2
5	101	7	0	0	5	0	3	0	0	0	3	2	0	1

Figure 8. A picture of the spread sheet that was used to analyze Olivia's observed discourse patterns.

Olivia only had three episodes that were coded with a WU-2 and no episodes that were coded with WU-3 or above. Because this is such a small sample of above average uses of student thinking it is difficult to draw any connections. In these three episodes Student Misconception, Mathematics and Assess all occur twice. Hence, I cannot conclude if these three reasons were positively affecting how well Olivia used student thinking or not. However, Length and Student Engagement or Disposition were above average for all three Episodes. Just as in the analysis of Megan's data Student Engagement or Disposition was not used because it conveys the same information as Length. Hence, Length seems to be the only viable predictor of Olvia's WU.

In Olivia's Length column 62 out of the 141 episodes were above average. Of these 62, three of them were WU-2. This means that when Olivia would choose to stay on a topic for longer than average, 5% of the time she would use student thinking in more productive ways. This percent of episodes of student thinking is obviously very small. This size is related to the fact that Olivia struggled with using student thinking in effective ways. Olivia was very consistent in using thinking. Unfortunately she was consistent in using student thinking by talking about it or using it as a trigger (B. E. Peterson & Leatham, 2009). Nevertheless, these results show that even for someone, like Olivia, who is struggling in using student thinking, if they take the time to focus a class discussion on a particular topic, they have a better chance (even if it is just a slightly better chance) of using student thinking in more productive ways.

Because of Olivia's limited number of WU-2's within the data, I compared all three episodes side by side to see if there were any other similarities. In all three episodes the students played a key role. In each episode the students seemed to beat Olivia in saying what she was going to tell them. Also, within these episodes the students demonstrated that they seemed to understand the mathematics at hand. The students demonstrated their mathematical understanding by correcting intentional mistakes made by the teacher or by correcting unintentional mistakes made by a peer. These students were willing to actively participate and help move the lesson along, which was not always the case. One fact that could help elucidate why the students were more willing to participate and why Olivia was more willing to peruse their thinking was that all of these episodes were centered around the idea of writing a linear equation in standard form (Ax+By=C). Maybe the students felt comfortable talking about this particular part of mathematics, or maybe Olivia knew that most of her students understood how to write equations in standard form so she was willing to deviate from her normal discourse pattern to try to extend her students thinking. Of either case I am unsure. However, I do not consider it to be a coincidence that all of the episodes where Olivia used her students thinking in slightly better ways happened to be covering the same mathematical topic. Also, in these episodes Olivia asked different questions than she would normally ask. Instead of asking simple questions, like "What do I do next?" or "So, what do I get on the other side of the equation?" she asked questions that focused on why the students acted the way they did instead of focusing on what they did. It seemed that asking better questions and turning the responsibility of thinking back to the students helped Olivia's students reason more deeply about mathematics.

Olivia's poor use of student thinking does not seem to come from poor reasons. Instead it seems to come from a faulty perspective of how to effectively use student thinking and of the results of that use. Olivia thought that repeating and expounding on student comments helped her students retain the information and engage in the lesson. Olivia never tried to gain any evidence that supported her in this use of student thinking. When Olivia would repeat or expound on student ideas her students would generally listen and they seemed to be paying attention. However, no more student thinking was elicited and Olivia assumed that all of her students understood. It seemed that Olivia needed to reflect on her use of student thinking by asking herself questions such as the following: How do I know when my students understand a concept? Generally, is one student saying one comment enough for me to assume that the rest of the class knows it? Am I more likely to explain a student's reasoning or would I have the students explain what they mean? Oliva's use of student thinking was problematic because it was not based off of her students' mathematical thinking. Instead it was based off of her own perspective.

CHAPTER 6: DISCUSSION

In this chapter I draw on the results presented in Chapter 5 to explicitly answer my research questions. While doing so I discuss how the results of this study tie to the literature.

Answering the Research Questions

When I began this research I had four research questions: (1) What decisions do STs make with respect to using their students' mathematical thinking? (2) Based on these decisions, how well do STs use their students' mathematical thinking? (3) What reasons are behind STs' decisions with respect to using student thinking? (4) How do the STs' reasons for the use of student thinking influence students' opportunity to learn? In the following four sections I explicitly answer these questions and tie them to the literature.

What decisions do STs make with respect to using their students' mathematical thinking?

The decisions that STs make with respect to using their students' mathematical thinking are only available to me as the researcher through viewing how the STs used the student thinking. The STs' decisions generally resulted in missed opportunities to better understand their students' thinking. Their uses of student thinking generally did not push students to make sense of the mathematics. Hence, the STs' decisions generally resulted in them using student thinking in ineffective ways.

Megan and Olivia actually used student thinking somewhat differently. Megan tended to elicit student thinking better than Olivia as seen by Megan's higher BU numbers. However, Megan and Olivia pushed on the student's current mathematical conceptions at about the same rate. This is seen with the similar numbers of ESM. Overall Megan and Olivia's WU levels were centered about WU-1 (student thinking was somewhat used). While Olivia was very consistent at marginally using student thinking, Megan had a more varied approach. Overall Megan and

Olivia tended to use student thinking in ineffective ways. Some of Megan's and Olivia's problematic uses of student thinking are discussed below.

A slightly different form of the I.R.E. Cycle was used by both STs, which has the appearance of using student thinking. This cycle would go in the following order: (1) The ST would ask a question; (2) A student would respond; (3) The ST would evaluate the response by repeating the student's response. If the ST stated the student's response in a matter-of-fact tone, then the response was correct and the lesson would move on. However, if the ST repeated the student's response as a question, then the student's response was incorrect and the correct answer needed to be given before the lesson could move on. This prevalent discourse pattern does not use student thinking effectively. Instead it seems to be related to Wood's (1998) funneling discourse pattern, in that the ST is the one who is doing the thinking while the students are just passive answer givers. However, what I see as being most problematic with this discourse pattern is that the STs actually think that they are effectively using their student's thinking, when they are not. Simply repeating a student comment is almost as ineffective as ignoring a student comment.

This modified I.R.E. Cycle is an example of what B. E. Peterson and Leatham (2009) referred to as "naïve use," which they described as STs believing that they were effectively using student thinking; in some naïve sense the STs were using the student thinking, however, the STs' use of student thinking was ineffective. Technically the STs could be considered to be using their students' thinking because they repeated what their student just said. However, this is a naïve use because the STs do not capitalize or build on the mathematical thinking of their students. This modified I.R.E. Cycle is also problematic because it removes the students' need to reason about mathematics. In all actuality, the students just have to be good guessers and have a partial understanding of what is being taught in order to get correct answers most of the time. Unfortunately this cycle helps students become the kinds of students who do poorly in mathematics classes. Namely they become guessers and only partially know the content.

Megan and Olivia both used routines in their teaching. Clark and Peterson (1986) and Borko and Shavelson (1990) all agreed that teaching routines affected teacher's interactive decisions. I concur with them that teaching routines seemed to make up a large part of Megan and Olivia's interactive decisions. These routines are most noticeable with how consistent the STs were with repeating student thinking to the class. For both STs, it seems that they have default teaching routines that must be changed in order to effectively use student thinking. Whether these default routines came from many years of poor observation or through their personal tendencies, these unproductive default routines are overwhelmingly present in their teaching practices. These practices especially effect how these STs used student thinking.

These default teaching routines seem to have these STs continually reacting to their students' thinking. The key aspect of cultivating or anticipating student thinking, however, is missing from their teaching. Consider the following hypothetical situation. If you were to play a game of basketball where you were not allowed to play offense and instead were only allowed to play defense, a lackluster game would result. By only reacting to your opponent's offense you are giving them control of the game. Your opponent will eventually stop pushing themselves. They will still play, but they will slow the pace of the game down, which slows your team down. All you will end up playing is a mediocre, one-sided game of basketball. This is the game these STs were playing. To play an exciting game you have to push your opponent as your opponent pushes you. You have to cultivate shots, set up plays, anticipate where your teammates will be and make the plays happen. This also goes for using student thinking. These STs were only reacting to student thinking. They were not pushing on their students' mathematics and, possibly as a result, their students seldom pushed back. This "defense mode" is the major reason why these STs tended to use student thinking in unproductive ways. To effectively use student thinking there is a give and take between the teacher and the students. By pushing on each other great plays can happen. I realize that the preceding analogy breaks down with scoring points and who wins the game. However, the main point is that the interaction of pushing on each other is what is important.

How well do STs use their students' mathematical thinking?

Megan and Olivia poorly used their students' mathematical thinking. They constantly repeated or revoiced student thinking. This use of student thinking was generally ineffective at getting the students to make mathematical connections and to reason deeply about mathematics. Most often how Megan and Olivia used student thinking was just about as effective as repeating student thinking or telling. Thus the classroom discourse that was produced did not generally give students the opportunity to learn.

Megan and Olivia often used a univocal discourse. Wood (1998) discussed a univocal discourse and compared it to a dialogic discourse. A univocal discourse conveys information much like a dictionary does, the definition to something is simply given. Where a dialogic discourse presents some information and has the reader has to make sense of the information. They have to make their own connections and conclusions. This discourse could be compared to a mystery novel. The STs in my research both tended to use a univocal discourse pattern, in that they did not generally create opportunities for their students to make sense of the mathematics. Instead, they often made sense of the mathematics and told students what they needed to know. However, a univocal discourse pattern was not strictly used. I saw several examples of dialogic discourse where the STs turned over reasoning about the mathematics to the students. These episodes of student thinking were coded WU-2 and WU-3. Hence, the better of the two discourses was seen, even if it was not as prevalent as would be desired.

Ball (1990) noted that many preservice mathematics teachers (both elementary and secondary) enter the field without a robust understanding of simple mathematics, like how to model the division of fractions. Not that modeling the division of fractions is easy, but these were future math teachers and it is something that they should be able to do. Likewise, I saw something very similar in this research. These two STs did not know how to use student thinking very effectively. Both STs seemed to be able to get students to talk about their mathematical ideas. However, it seemed that after the students' ideas were shared the STs did not really know what to do with them. So, they would repeat the ideas or merely talk about them. The STs would make sense of the mathematics so the students did not need to. Thus, these STs did not have a robust understanding of how to use student thinking.

What reasons are behind STs' decisions with respect to using student thinking?

Megan and Olivia's reasons for using student thinking were primarily focused on student engagement or student disposition. These engagement/disposition reasons dwarfed all of the other reasons combined. However, there were some other reasons categories that are of noteworthy size: mathematical-background, mathematical-student misconception, retention and not losing anyone. Thus, the STs' reasons for using student thinking were mainly associated with student engagement and disposition, while mathematical reasons were considerably smaller. The STs' reasons show what kinds of things the STs were concerned about while they were in the process of teaching. In this case both Megan and Olivia were predominately concerned about student engagement and disposition.

Marland (1979) pointed out that the six teachers who participated in her study often made inferences about why students were motivated, engaged, willing to work and about their students' current mathematical conceptions. However, none of the teachers checked with the students to see if their inferences were correct. Instead, the teachers assumed that their inferences (or reasons) were absolutely correct. This is exactly what I saw in my study. The two STs had reasons why how they were using student thinking and about how these uses should affect their students' learning. All of these inferences or reasons were positive, meaning that the STs were trying to accomplish something positive by their use of student thinking. However, the STs never checked to see if their actions were actually accomplishing what they intended. There were many cases where the STs' actions were counterproductive to their intentions. There needed to be some reflection and questioning about how they were teaching. They might ask themselves questions like, "Am I the conveyer of all knowledge in my class?" or "Do I require my students to think deeply about mathematics?" or "Can my students answer my questions without really thinking?"

How do the STs' reasons for the use of student thinking influence students' opportunity to learn?

As previously discussed the STs tended to ineffectively use student thinking, which tends not to give students an opportunity to learn. Most of the ineffective uses are connected to the STs trying to engage their students in the lesson through repeating student thinking. However, there were some more effective uses of student thinking that took place during this research. I say more effective because there was still room to improve on the use of student thinking but these cases are examples where the STs were able to use student thinking more effectively than they normally did, thus giving students more of an opportunity to learn. The reasons that were associated with these more effective uses of student thinking were mathematics-background and mathematics-student misconception. It seems that if the STs could focus more on these mathematical reasons while they teach they could more effectively use their students' thinking.

How the ST's reasons for using student thinking were associated with problematic uses of student thinking reminds me of Schoenfeld's (1988) article titled "When good teaching leads to bad results: The disasters of well-taught mathematics courses." However, I would title my article slightly differently than Schoenfeld did his. It would have to be something like, "When good reasons lead to bad results: The disasters of well-intentioned mathematics teaching decisions." I say this because it did not matter if the STs effectively used their student's thinking or if they failed at effectively using it, they always gave positive reasons for why they acted the way they did. For example, I never had a ST give a reason such as "I repeat what students say because I want my class to be boring and mundane." Instead, the STs would give reasons related to (naming only a few) student engagement, the mathematics at hand, student mathematical misconceptions, helping with retention, and not losing anyone.

All of these reasons given by the research subjects are things that all teachers should consider when teaching. However, even though these reasons were intended for good, there were problematic results. In this classroom generally students were not expected to reason deeply about mathematics. Instead the STs would often tell the students their reasoning for the mathematics. Student thinking was not used to help the students develop a more robust understanding of mathematics. Instead student thinking was generally used by repeating it or to change the topic of conversation. This mismatch from intentions and outcomes could come from the fact that these STs were using teaching routines to use their students' thinking. Hence these routines were used without the STs really thinking about how they were using student thinking or about the results of that use.

CONCLUSION

The following sections describe the implications of the study and directions for future research. The first section will cover implications or ideas that could help future STs to be more effective in using student thinking. The second section will cover ways to help STs overcome their problematic views. The third section will cover possibilities for future research and limitations of this study will be discussed.

Implications

Having studied STs' reasons behind their interactive decisions this research gives some insight in how to help future STs learn how to more effectively use their students' thinking. These finding will be particularly helpful to mathematics teacher educators, or those people who teach future mathematics teachers how to teach mathematics. This research gives three implications that could enable future mathematics teachers to learn how to better use their students' thinking: 1) Focus on spending the time to pursue student thinking. 2) Focus on how student thinking is used when there is a student misconception. 3) Focus on engaging students with the mathematics at hand.

STs should be encouraged to spend extra time on important topics. This research showed that when STs spent more time on a particular idea they tended to use student thinking in more effective ways. This does not mean that if the STs only focus on spending more time on each topic then they will magically use their students' thinking in productive ways. Instead, it is only a helpful part of what STs should focus on while teaching. It seems that when novice teachers run into students who appear not to want to think for themselves, that the STs quickly break down and give their students the answer. Hence, student thinking is not used effectively in this case. However, when STs expect their students to reason about mathematics and this is shown through persistence in following through on a particular subject, students are more likely to persist themselves. Thus, STs should not be afraid to spend a little extra time on a topic to try to help their students flesh out the main ideas.

This research showed that these STs used student thinking the best when they were focused on a student misconception. This means that these STs did have some teaching routines that effectively used and pushed on student thinking. However, both STs seem to buy into the fallacy that if they have not seen a misconception there must not be one. This is most likely untrue. Hence, one implication is that STs need to develop the ability to listen for misconceptions or to see potential misconceptions more often. By seeing potential misconceptions more often the STs would then be able to use their students' thinking more productively more often. So to better help these STs use student thinking the STs should focus on how they help their students with a misconception. The STs should also try to focus on perceiving and identifying possible student misconceptions more often. This type of discourse seems to better help students learn mathematics.

One of the reasons that the STs gave to use student thinking in productive ways was that the students needed to learn a piece of mathematics or that the mathematics at hand was interesting. Unfortunately these reasons were only given a small portion of the time, whereas reasons involving student engagement and disposition were given the majority of the time. The problem was that the STs were trying to engage their students with the lesson and not necessarily with the mathematics. By engaging in the lesson and not necessarily the mathematics I mean that the STs wanted their students to listen and pay attention instead of understand or make sense of the mathematics. Hence, another implication is that if the STs changed their ideas of student engagement to "understand" or "make sense of" they would be more able to engage their students with the mathematics at hand. By focusing on engaging their students in understanding or making sense of mathematics the STs would likely use their students' mathematical thinking more effectively.

It is important to note that how these STs used their students' thinking was not always poor. There was a lot to be done to improve their teaching. However, there were some good uses of student thinking. By focusing on what STs can already accomplish is probably the easiest way to get STs to effectively use student thinking more often. That is why I suggest having STs focus on the following three items. 1) spending the time to pursue student thinking; 2) reflecting on how student thinking is used when there is a misconception and seeing potential misconceptions more often; and 3) engaging the students with the mathematics at hand.

STs' Problematic Views and How They Can Improve

In the results chapter I brought up three places where at least one of the STs had good intentions but yet poor results followed. I think that this discrepancy comes because the STs have had insufficient time to think, review and become unsatisfied with their teaching in order to change the way they teach. For example recall that Olivia was trying to help her students with retention by repeating and talking about student thinking. I think that if Olivia thought about how she was trying to help with her students' retention she would eventually conclude that it is unproductive to re-tell a student something if they did not get it the first, tenth or one hundredth time. If this is the case (which it probably was) then her students' retention would not increase if she told them one more time. This is the major issue because Olivia has not seen a need to change how she is using student thinking. Hence, she has not changed how she uses student thinking like an expert teacher has. Olivia obviously has the tendency to explain her mathematical thinking. This tendency has a reason (generally student retention and or engagement) and how she addresses this reason is valid under her novice lens of teaching. However, it is not valid through an expert's lens of teaching because students will not retain or engage with the material because of how she is using the students' thinking.

One possible way for both STs to improve their use of student thinking would be to try to make the reasons that were associated with more effective use of student thinking to occur more often so the reasons could be more influential in their teaching. For example, Megan relies too much on reasons that cause her to ineffectively use her students' thinking. Through reflection, self observation, practice, etc. Megan could change her focus from ineffective reasons to use student thinking to effective ones. This change in focus could be part of the transition from novice teacher to expert teacher.

It seems that a ST's reason could be adapted or changed through a shift in the ST's lens of teaching mathematics. I see two options that could cause a ST to adapt their reasons. 1) The ST sees that what they are doing is ineffective or problematic and therefore stops using that particular reason. 2) The ST sees the need to replace a reason with something more effective, but the reason that is being replaced is not replaced in its entirety, just used less often. This second option could occur by the ST doing some sort of professional development in which they recently learned about a particularly new or interesting aspect of teaching or mathematics, so they decide to try to incorporate it into their teaching.

I saw an example of this type of reason adoption with Megan. While collecting data I filmed second and third periods. Olivia would teach second period then Megan would teach third period. Lunch was scheduled in between these two periods. One day while talking to both STs during lunch I shared an idea that fascinated me. It was that words that are used in math class are used outside of math classrooms in very different ways with very different definitions. For example, the words discrete, rational and irrational all have different meanings when used inside or outside of mathematics. Hence, I felt that mathematics is a language we have to teach to all of our students. Lunch ended shortly thereafter and Megan started teaching. In Megan's lesson the STs had planned to go over a word problem that said, "Write the algebraic expression of the product of $2v$ and u." Megan read the problem then asked, "What does the word 'product' mean?" A student responded with, "It is something you make or produce." Megan then had a small class discussion on what product means in mathematics. No doubt that Megan's adopted reason to bring up and peruse this student thinking was directly related to the discussion we had during lunch. This reason's adoption was confirmed in the follow up interview.

It is actually expected that novice teachers (STs) will have some misconceptions about how to use student thinking. That is exactly what I found. Both STs continually repeated their students' comments. They did this because they thought it helped with engaging their students in the lesson. Perhaps, if they stopped repeating their students and started having students do more of the thinking the students would be more engaged.

Not only was it expected that these STs would have some misconceptions about using student thinking, but it was also expected that periodically they would use their students' thinking in effective ways. Even if it did not occur very often, both STs were able to elicit and use student thinking in productive ways.

Future Research

This study has some limitations that leave room for future research. The size and scale of the study, the current discourse analysis, and other unexplored connections could be limiting factors.

This research only studied two STs during their student teaching experience at a Junior High School. This sample size is too small to draw any big overarching generalizations on STs' interactive decisions with respect to student mathematics thinking. Also, this study cannot make any generalizations towards BYU's Mathematics Education Program because I only studied two of the fourteen or so who were student teaching in secondary mathematics in that particular semester. The size of this study limits any generalizations in these two categories.

In my mind, this study did a good job in analyzing the positive aspects of the STs' discourse patterns. It was built to find out if the STs created opportunities to make sense of their students' thinking. This was called Better Understand, or BU. This discourse analysis also was built to find out if the students were building off of other's thinking. This was called Evidence of Student Mathematics, or ESM. The third portion of this discourse analysis was built to analyze how well the STs used the student thinking. This was called Well Used, or WU. BU, ESM and WU seemed to accomplish that which they were built to do. However, this study could be improved by adding to the discourse analysis a couple of quantifiers that would numerate problematic ways to use student thinking. BU and ESM focus on the positive uses of student thinking; however, only focusing on positive discourse patterns hides some of the problematic uses of student thinking, such as when a ST ignores a student comment. This "use" of student thinking is problematic, but the present discourse analysis generally does not catch such instances because they are often embedded within other episodes of student thinking. By saying this I mean that if during an episode of student thinking the ST ignored one student comment, there were probably other student comments that the ST used in some way. In this case the ST mostly used their student's thinking; hence the WU coding would not reflect the one problematic use of student thinking. The only way for the current discourse to catch a problematic use of student thinking is for that use to be isolated outside of any other episode of student thinking. Hence the problematic use could constitute its own episode of student thinking. This episode would then be coded as a WU-0. Granted the discourse analysis WU did show some problematic uses of student thinking, but it would be better to quantify problematic uses of student thinking in the same way BU and ESM quantify positive discourse patterns. If the problematic uses of student thinking could be better quantified then it would help paint a better picture of why STs use student thinking in problematic ways.

One other area to explore would be to see how the cooperating teacher's use of student thinking affects how the STs use student thinking. I bring this up because the cooperating teacher in my study taught very traditionally, even though he had tried teaching in a reform-oriented manner at one time. At the time of this study he had reverted back to a traditional telling model with a lot of homework practice. Hence, I feel that my research subjects could have been affected by their cooperating teacher's predisposition not to use student thinking. It would be interesting to see if there is a connection there. Unfortunately the scope of this research did not cover looking at the cooperating teacher's influence on the STs.

By studying STs' interactive decisions with respect to student mathematics thinking I hoped to better understand how STs use their students' mathematical thinking, and how the STs' reasons for using student thinking help or hinder their use of student mathematics thinking. Through this research I found that my test subjects tended to use student thinking with the best of intentions, but in unproductive ways. However, there were a few episodes of student thinking where the STs succeeded in productively using student thinking. These episodes were longer than average and the STs' reasons were focused on addressing a student's misconception, or addressing a particular aspect of mathematics. These findings imply that by helping STs focusing on what they can already accomplish is probably the easiest way to get STs to effectively use student thinking more often. These findings are important for mathematics teacher educators to be able to better help their future STs prepare to effectively use student thinking in the classroom.

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APPENDIX A

Example of Pre-lesson Survey

Lesson Date:

What (if any) are your unit goals that pertain to this lesson?

What is the topic of this lesson?

What are your lesson goals?

Are you anticipating any student comments or questions that may come up during this lesson? If so, what are they?

Please attach a copy of any written lesson plan.

APPENDIX B

Post-Lesson Interview Protocol

Each episode of student thinking is coded as do not use, teacher talk, run with or other before the interview. I will then ask the following set of questions for each type of use.

Sample interview protocol:

1) In the segment of your lesson I just described, you decided to ________________________. Why did you choose to_____________________?
   Were you considering anything else as an alternative course of action? If so, what other alternatives were you considering?
2) Why did you decide to use the student thinking in this particular way?
3) If you were put into a similar situation would you still react the same way?
   Do not Use: Are there circumstances where you might choose to use this comment?

Teacher Talk/Run With: Are there circumstances where you would use this comment differently?

APPENDIX C

Example of Field Notes

FIELD NOTES

Date	
Student Teacher	
Lesson Goals/ Other Goals	

Time of Episode	Elapsed Time	----------------

Context	Student Remark	Teacher Response
	Time Recording Started	
Questions:		
Time of Episode	Elapsed Time	----------------

Context	Student Remark	Teacher Response
Questions:		
Time of Episode	Elapsed Time	----------------

Context	Student Remark	Teacher Response

________	________	________
Questions:____		

APPENDIX D

Example of Final Interview Protocol

Interview Protocol:
The purpose of the next few questions is to find out some of your teaching beliefs. There is no right or wrong answer. I just want to know what you think, ok?
1) As a math teacher what do you think it means to have a good class discussion?
-Could you share a hypothetical (or real) example of a good class discussion?
- In this discussion, what do students do? Or what is normal for the students?
- In this discussion, what does the teacher do? Or what is normal for the teacher?
-On a scale from one to ten, one being the low and ten being the high, how important is class discussion to your teaching? Please explain why.
2) What do you think of when I say the phrase: "The math teacher used her student's thinking."?
-How would a teacher use her student's thinking?
-(if an example is not given) Could you give a hypothetical (or real) example of a teacher using her student's thinking?
-Do you, as a math teacher, use your student's thinking?
-Please give an example of you using your student's thinking.
-On a scale from one to ten, one being the low and ten being the high, how important is it for math teachers to use their student's thinking? Please explain why.
3) I know that besides teaching Algebra A, you also taught Geometry and possibly Algebra I as part of your student teaching. When we first met we decided that Algebra A was the best choice of classes for me to observe because it had the easiest schedule to work with. I think the main reasons why we chose it were you had a prep hour after, the class didn't have a set timeline of when topics had to be covered and I didn't have to come and observe first hour. Now after the fact we might have changed our minds. I know that one of you mentioned in passing that we might have picked the wrong class for me to observe, possibly because I didn't get to observe your best lessons. If you could choose again, would you choose algebra A for me to observe? Why?
-What were some of the difficulties you faced in teaching Algebra A?
-Do you feel that some of the difficulty you experienced in Algebra A was related to the mathematics you had to cover as part of the course?
-Do you feel that some of the difficulty you experienced in Algebra A was related to your students and their mathematical abilities?
-What were your mathematical expectations of your Algebra A students? (What did you expect your Algebra A students to be able to handle or do mathematically on any given day?)
-For the most part were your Algebra A students able to reason deeply about mathematics?
-(yes) Do you feel like you helped your students reason deeply about mathematics?
-(yes) Can you give an example of how you helped your students reason deeply about mathematics?
-(no) Why do you feel this way?
-(no) Why do you think they were unable to deeply reason about mathematics?